Towards a Self-Calibratable Gravimeter on a Chip: Theoretical Study

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ABSTRACT

In this paper, we provide a theoretical study of our proposed gravimeter on a chip. A gravimeter is a device used to measure gravity or changes in gravity. There are several kinds of conventional gravimeters: pendulum, free falling body, and spring gravimeters. They are all large, expensive, delicate, and require an external reference for calibration. What is new and different about our proposed gravimeter is its micro-scaled size which increases portability, robustness, and lowers it costs; and its ability to self-calibrate on chip, which increases its autonomy. Gravimeters are most often used in geophysical applications such as measuring gravitational fields for navigation, oil exploration, gravity gradiometry, earthquake detection, and possible earthquake prediction. Precisions of such gravimetry can require measurement uncertainties on the order of $20\mu Gal = 20\times10^{-8}\, m/s^2$. In our study we present the self-calibration method of a proposed MEMS gravimeters and we predict its ability to achieve the necessary accuracy and precision needed for its use as a gravimeter or sub-micro-G accelerometer. For practical reasons, our MEMS design to adhere to the design constraints of a standard SOI foundry process.

Keywords: Gravity, Accelerometer, Gravimeter on a chip, MEMS, Self-calibration

1 INTRODUCTION

A gravimeter is a device used to measure gravity or changes in gravity. They are often referred to absolute and relative gravimeters respectively. Gravimeters have found application in geophysical and metrological areas such in navigation, oil exploration, gravity gradiometry, earthquake detection [1], and possible earthquake prediction [2]. Measurement resolution that is often required in the above geophysical applications to resolve spatial gravity variations is $\sim 20\mu Gal = 20\times10^{-8}\, m/s^2$ ($1 Gal = 0.01\, m/s^2$). However, the time rate of gravitational change for many crustal deformation processes is on the order of $1\mu Gal$ per year [3]. Gravimetry is also used in a number of metrological measurements such as the calibrations of load cell for mechanical force standards [3]. Desirable attributes for gravimeters are smaller size, lower cost, increased robustness, and increased resolution. Decreasing their size increases their portability. Lowering their costs allows a larger number of them to be deployed simultaneously for finer spatial resolution. Improving their robustness to changes in temperature, age, and handling improves their reliability or repeatability. And improved accuracy and resolution increase confidence in measurement.

We propose a new generation of gravimeters that are about a 100 times smaller (meter-size to centimeter-size), 1000 times lower in cost ($500k-$100k to ~$50), just as accurate and precise, and with the unique ability to self-calibrate at any moment. Microfabrication reduces the size and costs of such a device by being able to batch fabricate a multitude of microscale devices simultaneously. And our self-calibration technology is expected to allow our devices to recalibrate after experiencing harsh environmental changes or long-term dormancy.

We show some conventional gravimeters in Fig. 1. A pendulum gravimeter (Fig. 1a) is used to measure absolute gravity by measuring its length, maximum angle, and period of oscillation [4]. Its accuracy depends on the external calibration of such quantities. A free falling body gravimeter (Fig. 1b) is used to measure absolute gravity by measuring the acceleration of a free falling mirror in a vacuum by measuring the time for laser pulses to return from the falling mirror [3]. It requires external calibration of the laser pulse timing system. A spring gravimeter (Fig. 1c) is used to measure relative gravity by using a spring supported mass to measure a change in static deflection between a reference gravitational position and a test gravitational position [5]. It requires external calibration of spring stiffness, proof mass, and displacement. A depiction of a microscale device for measuring sub-micro-G accelerations ($< \mu G = \mu\, 9.80665\, m/s^2$) is shown in Fig. 1d. It requires the external calibration due to a known acceleration. With respect to calibration, we propose a MEMS that is able to measure its own stiffness, displacement, and mass, and is useful for absolute or relative gravimetry, or sub-micro-G accelerometry.

The rest of the paper is organized as follows. In Section 2 we describe our unique self-calibration method. In Section 3 we show describe our proposed MEMS gravimeter and predict its performance. And in Section 4 summarize our study. We provide our nomenclature in Table 1.

TABLE 1: Nomenclature

h	Thickness of the device layer (unknown)
g	Gap between comb fingers (unknown)
ε	Permittivity of the medium (unknown)
β	Capacitance correction factor (unknown)
L	Initial finger overlap (unknown)
C, C^P	Capacitance (measured)
Δ, δ	Difference and uncertainty (measured)
x	Comb drive displacement (measured)
F	Comb drive force (measured)
k	System stiffness (measured)
gap	Gap stop size (measured)
Ψ	Comb drive constant (measured)
Δgap	Layout-to-fabrication (measured)
V	Applied voltage (known)
N	Number of comb fingers (known)
n	$n = gap_{2,layout}/gap_{1,layout} \neq 1$ (known)
gap_{layout}	Layout gap (known)
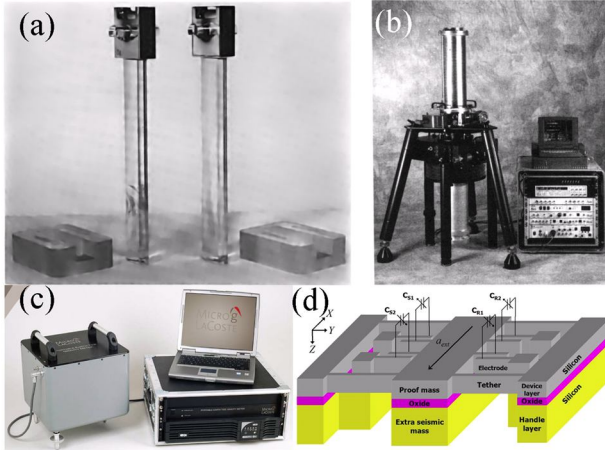
Figure 1: Some conventional gravimeters. (a) Pendulum gravimeter [4], (b) Free fall gravimeter [3], (c) Spring gravimeter [5], and (d) Sub-micro-G accelerometer [1].

2 SELF-CALIBRATION
2.1 Change from layout to fabrication

Electro micro metrology (EMM) is an accurate, precise, and practical method for extracting effective mechanical measurements of MEMS [8]. The method of EMM that we implement here begins by using two unequal gaps to determine the difference in gap geometry between layout and fabrication. These gap stops establish a means of equating a well-defined distance in terms of change in capacitance. In subsequent sections, we use this gap-stop to facilitate measurements of comb drive displacement, force, and system stiffness.

We show our proposed self-calibratable MEMS gravimeter in Figure 1. Two unequal gaps are related by $gap_{2,Layout} = n\, gap_{1,Layout}$. They are needed to provide two necessary measurements to eliminate the unknown properties listed in Table 1 as follows.

Using differential capacitive sensing, measurements at zero-state and upon closing gap_1 and gap_2 by applying enough actuation, voltage may be expressed have the forms as

$$\Delta C_1 = -4N\,\beta\,\varepsilon\,h\left(gap_{1,layout} + \Delta gap\right)\big/g, \tag{1}$$

where define $\Delta gap \equiv gap_1 - gap_{1,layout}$, and parasitics cancel in the difference. Similarly, closing the second gap yields

$$\Delta C_2 = 4N\,\beta\,\varepsilon\,h\left(n\,gap_{1,layout} + \Delta gap\right)\big/g. \tag{2}$$

We eliminate the unknowns by taking the ratio of (1) to (2) and solve for the *measurement* of the change in gap-stop from layout-to-fabrication as

$$\Delta gap = -gap_{1,layout}\left(n\Delta C_1 + \Delta C_2\right)\big/\left(\Delta C_1 + \Delta C_2\right). \tag{3}$$

2.2 Calibrating Displacement, Stiffness, Mass

Once ΔC_1 and Δgap are measured, the comb drive is calibrated. We measure the comb drive constant as

$$\Psi \equiv \Delta C_1 \big/\left(gap_{1,layout} + \Delta gap\right) = \Delta C_1/gap_1, \tag{4}$$

where Ψ is the unique quantity $4N\,\beta\,\varepsilon\,h/g$ expressed in the previous section.

Displacement. That is, Ψ is the ratio of the change in capacitance to traverse a gap-stop distance to that distance. This ratio is applies to any intermediate displacement $x \leq gap_1$ and corresponding change in capacitance ΔC. We may therefore measure displacement as

$$\Psi \equiv \Delta C_1/gap_1 = \Delta C/\Delta x \;\Rightarrow\; \Delta x = \Psi^{-1}\Delta C. \tag{5}$$

Electrostatic force. When applied to comb drives within their large linear operating range, the partial derivatives can be replaced by differences. The electrostatic force is measured as

$$F \equiv \tfrac{1}{2}V^2 \partial C\big/\partial x = \tfrac{1}{2}V^2 \Delta C\big/\Delta x = \tfrac{1}{2}V^2\Psi. \tag{6}$$

where we have substituted our measured comb drive constant from (4). It is important to note that our force in (6) accounts for fringing fields and accommodates some non-ideal asymmetric geometries in the comb drive due to process variations.

Stiffness. From measurements of displacement and force, system stiffness is defined as their ratio as
$$k \equiv F/\Delta x = \tfrac{1}{2}\Psi^2 V^2\big/\Delta C \tag{7}$$
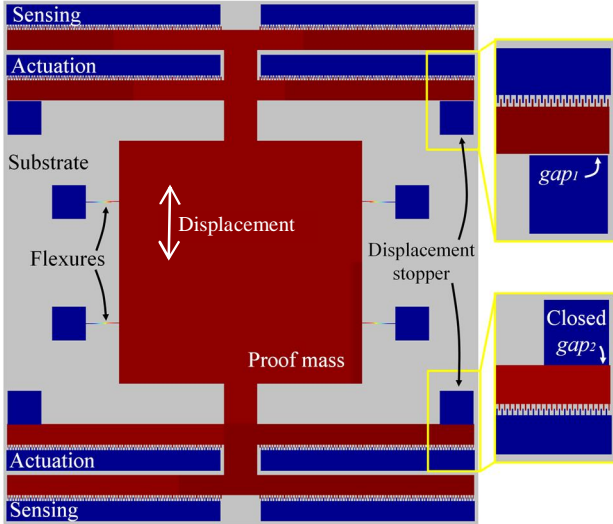
Figure 2: Self-calibratable gravimeter. Color mapped displacement is shown. Actuation comb drives have closed *gap2*. The substrate underneath the proof mass is backsided etched to release the proofmass. Design adheres to the SOIMUMPs process [6].

which is able to account for large nonlinear deflections. the quantity $V^2/\Delta C$ in (7) is nearly constant for small deflections, but is expected to increase for large deflections.

Mass. From measurements of stiffness from (7) and resonance ω_0, system mass can be measured as

$$m = k/\omega_0^2 , \qquad (8)$$

where ω_0 is not the displacement resonance that is affected by damping, but the velocity resonance that is independent of damping and equal to the undamped displacement frequency.

2.3   Measuring uncertainties

One method for measuring uncertainties is done by taking a multitude of measurements and computing the standard deviation in measurement from the computed average. As the number of measurements increase, the smaller the standard deviation becomes. If taking a large number of measurements is impractical, we propose a more efficient method of measuring uncertainties due to a single measurement as follows.

With respect to the above analyses, electrical uncertainties in the measured capacitance δC and voltage δV produce corresponding mechanical uncertainties in displacement δx, force δF, mass δm, and stiffness δk. To determine such uncertainties, we rewrite all quantities of capacitance and voltage in the above analyses as $\Delta C \rightarrow \Delta C + \delta C$ and $\Delta V \rightarrow \Delta V + \delta V$. We then identify the first order terms of their multivariate Taylor expansions as the mechanical uncertainties. The uncertainties in displacement, force, stiffness, and mass are:

$$\delta x = \left(gap_{1,layout}\left(1-n\right)\frac{\Delta C_1 + \Delta C_2 - 2\Delta C}{\left(\Delta C_1 + \Delta C_2\right)^2} \right)\delta C \qquad (9)$$

$$\delta F = \left(\frac{V^2}{gap_{1,layout}\left(1-n\right)} \right)\delta C + \left(\frac{\left(\Delta C_1 + \Delta C_2\right)V}{gap_{1,layout}\left(1-n\right)} \right)\delta V \quad (10)$$

$$\delta k = \left(\frac{-\left(\Delta C_1 + \Delta C_2\right)\left(\Delta C_1 + \Delta C_2 - 4\Delta C\right)V^2}{2\left(n-1\right)^2 \Delta C^2 gap_{1,layout}^2} \right)\delta C$$
$$+ \left(\frac{\left(\Delta C_1 + \Delta C_2\right)^2 V}{\left(n-1\right)^2 \Delta C\, gap_{1,layout}^2} \right)\delta V \qquad (11)$$

and

$$\delta m = \frac{1}{\omega_0^2}\delta k + \frac{2k}{\omega_0^3}\delta\omega . \qquad (12)$$

3   PERFORMANCE PREDICTION OF A GRAVIMETER ON A CHIP

In this section we use the EMM results from Section 2 as a design factor in predicting the desired resolution of a MEMS gravimeter. That is, we find the necessary uncertainties in capacitance, voltage, and frequency to predict the precision in our device's measurement of gravitational acceleration. We parameterize flexure length. Other parameters such as mass, number of comb fingers, finger overlap, flexure width, layer thickness, etc., also affect precision but are not presented here. We choose the following parameters: 1000 comb fingers total, $2\mu m$ gap between each finger, $2\mu m$ flexure width, $3500\mu m$-squared proof mass, and single crystal silicon material.

Design issues. Besides the abovementioned parameters, other issues that a designer might consider are the sizes of the gap-stops, the range of gravitational forces, and the comb drive levitation effect.

Gravitational acceleration acting on one of our proposed MEMS gravimeter designs is identified in Fig. 2. The constraints on the geometry and material properties of the MEMS follow the $25\mu m$-thick SOIMUMPs design rules [6]. The anchors near the comb drives provide the required gap-stops for self-calibration as discussed in Section 2. The size of these gaps must be necessarily larger than the normal operating displacements due to the expected range of gravitational forces. On the other hand, the gaps must not be so large that an unusually-large voltage would be required to close and calibrate the device.

For the type of EMM analysis presented in Section 2, we suggest that the translation of the comb drive remains in plane. Comb drive levitation can cause a slight out-of-plane deflection. Such levitation is produced when there is an asymmetric distribution of surface charge about the comb fingers. This is usually due to the close proximity of the underlying substrate. We have found that implementing a backside etch underneath comb drives can help circumvent this levitation effect [7].
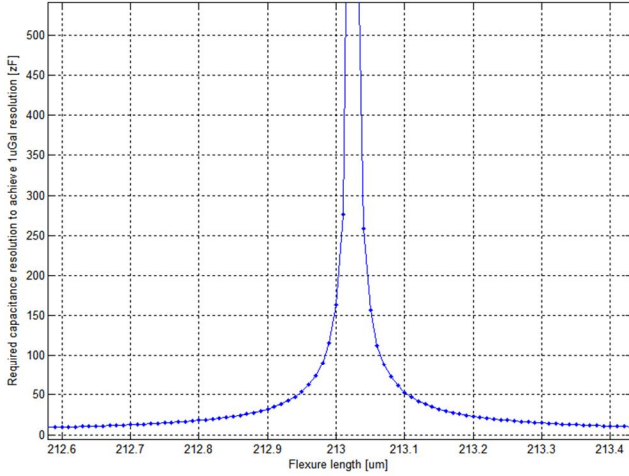
Figure 3: Uncertainty in capacitance δC **as a function of flexure length** L**.** The y-axis (δC) ranges from 0 to 575 zeptofarads, and the x-axis (L) ranges from 212.6 to 213.4 microns. What is interesting is the possible cancelation of the effect of uncertainty in capacitance as seen by the peak. However, the peak occurs over a small range < 0.1 microns, which does not allow for much process variation in geometry. Widening this width of this curve and or creating designs that are more insensitive to process variation is desirable.

Results. To predict the uncertainty in measurement of our proposed MEMS gravimeter, we express measurements as follows. Nominal measurement of gravitational acceleration is $g = k\,x/m$. Uncertainty in measurement yields $g + \delta g = (k + \delta k)(x + \delta x)/(m + \delta m)$. Substituting uncertainties (9), (11), (12), a multivariate Taylor yields

$$\delta g = \left(\frac{G\,g}{2\,gap_1\,h(n-1)\,N\,\varepsilon} - \frac{g\,E\,w^3}{N\,\varepsilon\,m\,L^3} \right)\delta C$$
$$+ \left(\frac{G}{L^{3/2}}\sqrt{\frac{m}{E\,w^3\,h}} \right)\delta\omega \qquad (13)$$

which shows that the resolution of the gravitational acceleration depends on the uncertainties of δC and $\delta\omega$.

For a case study of (13), let's assume typical measurement values for the following quantities: stiffness $k = 4Ehw^3/L^3$ based on flexure length L that we sweep below, mass $m = density \times volume$, $x = mg/k$, ΔC based on x, and ω_0 from (8). As previously mentioned a $1-20\mu Gal$ resolution is desirable. By constraining (13) such that $\delta g = 1\mu Gal$, in Figures 3 and 4 we plot δC and $\delta\omega$ as functions of flexure length L (L changes stiffness).

In Figure 3, we find that it may be possible through design to eliminate the sensitivity to uncertainty in capacitance. This is seen as the peak in the plot, were the uncertainty can be extremely large; and can be seen in (13) within the parenthetical expression which can possibly cancel depending on the choice of design parameters.
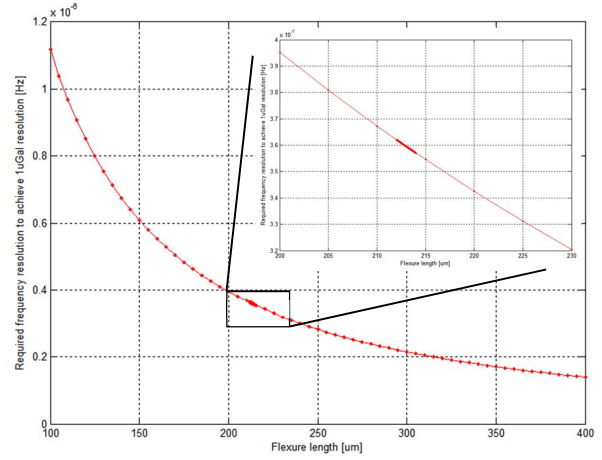
Figure 4: Uncertainty in frequency $\delta\omega$ **as a function of flexure length** L**.** The y-axis ($\delta\omega$) ranges from 0 to 1.6 micro-Hertz, and the x-axis (L) ranges from 100 to 400 microns, with an inset of 212.6 to 213.4 microns. Inset follows Figure 3.

In Figure 4, we find that the uncertainty in frequency plays a very significant and sensitive roll. Since the sensitivity with respect to frequency is large, the uncertainty in frequency must be quite small such that a $\delta g = 1\mu Gal$ resolution is achieved. In our particular test case, a resolution of about 1 to 10 μHz.

It is important to note that we have not optimized the geometry of our structure. Although we have explored the resolution in measuring gravity as a function of flexure length, we have not explored the effect of varying mass, gap between fingers, size of gap-stops, number of fingers, flexure width, etc.

4 CONCLUSION

In this paper, we present a theoretical study of our proposed gravimeter on a chip. We examine a test case were we determine what uncertainties in electrical measurands are required to achieve the desired uncertainty in gravitational acceleration. We find that we can eliminate the uncertainty due to voltage, and that we can possibly effectively eliminate the uncertainty in capacitance. This leaves the uncertainty in frequency, which should be on the order of micro-Hertz. The design of our test case was not optimized, where we only parameterized flexure length, and we adhered to SOIMUMPs design rules.

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