The Concatenation of the Concurrent Self-replication and Self-organization Processes

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ABSTRACT

In the paper the concatenation of the concurrently running self-replication processes of objects, and concurrently running self-replication and self-organization processes which besides of the multi-stage product synthesis (*gluey matrix* technology) are basis of the *Molecular Nanotechnology of Direct Product Fabrication*, has been analyzed and discussed. Analysis of the interdependent self-replication processes and also, the mutual self-replication and self-organization processes basing on the growth functions, provides facilities for determination of the control of these processes in the sense of stabilization, process extinguishing, and avalanche proliferation. In the paper, the numerical example has been presented.

Keywords: concurrent processes, concatenation, molecular simulation, self-organization, self-replication

1 INTRODUCTION

It is possible to create technical nanosystems of informatics which will be realizing like in the case of biological systems the sets of technological operations which material products are coming into existence and so will be taken to manufacturing objects and products on the basis of the molecular nanotechnology. It leads to the three problems connected with the realization of creating the technical nanosystems of informatics of the direct product fabrication and it concerns: the nanosystems of informatics, the self-replication and the self-organization processes.

In *nanosystems of informatics* the process of manufacturing is based on the extorted connecting of elements of the structure to the form of intermediate product for desirable product and modifications of the intermediate product created till the obtainment of the appropriate final product.

The *self-replication* is being realized in technical systems of informatics through the replication of molecular objects together with molecular programs stored in them. The strategy of construction of products and objects in technical systems of informatics can use the general strategy appearing in biological systems in a form of the embryonic development or can implement the processes of *self-organization*.

In the paper the concatenation of the concurrent self-replication and self-organization processes is discussed.

2 SELF-REPLICATION PROCESSES

Let's assume that there is a set of determined elements or *environment*. Let's assume that the energy provided from outside ensures mobility of the elements and as a result, free movement conditions are fulfilled in the environment area. An *object* means a part of the environment selected in such a way that the exchange of elements between the environment and the object is possible provided that the determined conditions are fulfilled. Due to the above, the object can gather elements necessary for its development and can develop in accordance with its internal program. If in a consequence of the object development, the partition of other objects, which are able to continue successive partition, and descendant objects, which are able to continue their independent development processes, takes place; such a phenomenon is called a *self-replication process*.

2.1 Growth Function in Replication Process

Let us define the measure of a time k to be the integer while the time unit we denote by the symbol 1. Considering the spontaneous self-replication process, we describe it in such a manner, that the number of elements created after the time k is described by the growth function $S(k)$ receiving integer value for dimensionless argument $k = 0, 1, 2, 3, ...$ which describes the consecutive number of time interval. The two basic possible types of self-replication may arise: linear and avalanche.

2.2 Linear Self-replication

Let $S(k)$ be the growth function of self-replication process described by

$$S(k+1) = S(k) + A, \qquad (1)$$

with initial condition $S(0) = B$.

Symbol A denotes a number of elements about the same properties, participating in the process, for which a total number of elements is rising in the k time interval, while B is determining the number of elements in the initial moment of the self-replication process. We will make an assumption, that if $S(k)$ in (1) is zero, then the process stops (and similarly in different kinds of the self-replication).

The growth defined by the function $S(k) = 1 + k$ in Eq. (1), we will call as a *linear growth*.

Example 1

Let be $A = 1$ and $B = 1$; the total number of elements $S(k)$ after time k is

$$S(k) = k + 1, \text{ or more precisely } \lfloor S(k) \rfloor = 1 + \lfloor k \rfloor, \qquad (2)$$

where a symbol $\lfloor \rfloor$ denotes a step function [5], which value is changing only for integer values of argument and between them the value of function is staying constant.

2.3 Avalanche Self-replication

Let the growth function $\int S(k)$ of the self-replication process be determined by equation:

$$\int S(k+1) = \int A(n) \cdot S(k), \quad (3)$$

with initial condition $S(0) = B$. Symbol $A(n)$ denotes a function of argument n, determining a number of elements created in the time (*avalanche growth*) unit about the same properties as properties of the element generating them.

Example 2

Let be $A(n) = n+1$ and $B = 1$; the total number of elements $\int S(k)$ after time k is:

$$\int S(k) = (n+1)\int S(k-1) = (n+1)(n+1)\int S(k-2) = \underbrace{(n+1)(n+1) \quad \ldots \quad (n+1)}_{k} \cdot \int S(0)$$

that is to say

$$\int S(k) = \int (n+1)^k. \quad (4)$$

Resolving (4) for e.g. $n = 2$ we obtain $\int S(k) = \int 3^k$.

3 SELF-ORGANIZATION PROCESSES

Let us assume that there is a set of N elements. The elements of a set N stay in a chaotic movement and have a such a property, that if will be found in a suitable mutual position, they are connecting together and are already permanently staying in such a position forming the strand as a result of the *self-organization* process. If the set contains N of elements, than the length of longest possible strand amounts d_N. The state, when the all N elements of the set connects together forming the strand about the d_N length, is one of possible steady states finishing this elements' connecting process (we will assume that the closed strands does not appear [3]).

Let the step function $\int f(x)$ determines the number of unconnected elements of a set N in the x moment. Let the measure of the x time will always be an integer positive numbers. We will mark the elementary unit of the time with symbol 1. We will assume that only one connection is created between the elements of a set N in the elementary time 1, so for the initial condition $\int f(0) = N$ we may write

$$\int f(x) = N - \int x. \quad (5)$$

Let us go from the x time appointed with moments of connections of elements, to the real time k (for the sake of notation simplification in here the symbol k will be also the positive integer number, determining the number of time interval). Depending on process, in general we may write

$$k_{i+1} = P(k_i), \qquad \text{where } i = 0,\ 1,\ 2,\ 3,\ \ldots\ ,$$

where $P(.)$ depends on process, e.g. for

$$k_{i+1} = k_i + (1+i), \quad (6)$$

(growing up of intervals with arithmetic progression [3, 8]), basing on (5) and (6) we obtain (for proper initial condition) for the rise of the strand length $\int L(k)$

$$\int L(k) = \int \tfrac{1}{2}\left(\sqrt{1+8k} - 1\right). \quad (7)$$

4 CONCATENATION OF THE SELF-REPLICATION PROCESSES

In general, through the term *concatenation* we understand the interdependencies occurring between processes running concurrently.

We consider e.g. two self-replication processes determined by the growth function $\int S(.)$ and $\int T(.)$, which can be written using the following notation:

$$\int S(k_{i+1}) = \int F_1[\int S(k_i), k_i], \quad (8)$$

with initial condition $S(k_0) = W_1$

and

$$\int T(k_{i+1}) = \int F_2[\int T(k_i), k_i], \quad (9)$$

with initial condition $T(k_0) = W_2$,

with that:

$$k_{i+1} = k_i + 1, \qquad i = 1,\ 2,\ 3,\ \ldots \quad (10)$$

$$k_0 = 0. \quad (11)$$

The functions $\int F_1(.)$ and $\int F_2(.)$ describing processes (8) and (9), fulfill the relations:

$$\int \Delta S = \int S(k_{i+1}) - \int S(k_i) \geq 0 \quad (12a)$$

$$\int \Delta T = \int T(k_{i+1}) - \int T(k_i) \geq 0, \quad (12b)$$

for all i, that is to say the growths functions of these processes are non-decreasing, i.e. the number of created elements in the process increases, may be stopped, but in any case cannot decrease. It can be noticed, we analyze the processes determined by the non-decreasing growth functions.

Let's take note, that in general case the time interval number defined in (10), can be incremented with value non-equal 1, similarly, the number of initial moment can be different than 0 as in Eq. (11). The introduction of this manner of denotation of the arguments in (8) and (9), allows to consider much wider class of processes in comparing with simple formalism introduced earlier in Eqs. (1) and (3).

In further, we will find the relation between growth functions of the processes $\int S(.)$ and $\int T(.)$, and a growth function of the process $\int R(.)$, establishing concatenation of processes $\int S(.)$ and $\int T(.)$ defined as follow.

A division of a set of elements $\int R(k_i)$ in the time interval k_i for further divisions inside the running self-replication process, is defined by the parameter $0 < \alpha < 1$ in the following manner:

for continuation of a process $\int S(.)$

$$\int S(k_i) = Ent\ \{\alpha \cdot \int R(k_i)\}, \quad (13)$$

for continuation of a process $\int T(.)$, with respect to (13), one may write:

$$\int T(k_i) = \int R(k_i) - Ent\ \{\alpha \cdot \int R(k_i)\}. \quad (14)$$

Symbol $Ent\{.\}$ denotes the *entier* function, sometimes expressed as $E(.)$ or $[.]$.

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Parameter α may receive all values in the interval (0; 1) with excluding the interval endpoints i.e. $\alpha = 0$ and $\alpha = 1$, because these are the particular cases (independent, alternative process realization of $\int S(.)$ or $\int T(.)$).

Definition 1

The growth function $\int R(k_i)$, caused by concatenation of the growth processes $\int S(k_i)$ and $\int T(k_i)$, is defined on the set of discrete argument k_i in the following manner:

$$\int R(k_{i+1}) = \int F_1[\int S(k_i), k_i] + \int F_2[\int T(k_i), k_i],$$
$$\int S(k_{i+1}) = Ent\{\alpha \cdot \int R(k_{i+1})\}, \quad (15)$$
$$\int T(k_{i+1}) = \int R(k_{i+1}) - \int S(k_{i+1}),$$

for initial condition:

$R(k_0) = A$ (A integer),

$S(k_0) = Ent\{\alpha \cdot R(k_0)\}$, $T(k_0) = R(k_0) - S(k_0)$,

$k_{i+1} = k_i + 1$ for $i = 0, 1, 2, 3, \ldots$ and $k_0 = 0$. ■

Example 3

Let's consider the concatenation of two self-replication processes: linear self-replication (1) and an avalanche (3). The goal of the analysis is to find the growth function $\int R(.)$ of the process establishing the concatenation of the two self-replication processes characterized by the process parameters the same as in the Examples 1 and 2.

So, we consider two processes, for which in case, when they are running independently, their description has the following form:

Process I:

$$\int S(k_{i+1}) = \int S(k_i) + A, \quad (16)$$

Process II:

$$\int T(k_{i+1}) = A(n) \cdot \int T(k_i), \quad (17)$$

with sufficient initial conditions.

Let's assume, that the concatenation of the processes (16) and (17) is performed with the coefficient $\alpha = 0.3$ (Equation (13)). Accepting the particular parameters in Eqs. (16) and (17), the same as in Examples 1 and 2, i.e. $A = 1$, $n = 2$ and $A(n) = n + 1$, we get the process $\int R(.)$ with initial condition $R(k_0) = B = 1$, generated by the concurrently running processes I and II. The process $\int R(.)$ defined by the dependencies (8), (9) and (15) can be developed using the set of following equations:

$$\int R(k_{i+1}) = \int S(k_i) + 1 + 3 \cdot \int T(k_i),$$
$$\int S(k_{i+1}) = Ent\{0.3 \cdot \int R(k_{i+1})\}, \quad (18)$$
$$\int T(k_{i+1}) = \int R(k_{i+1}) - \int S(k_{i+1}),$$

with initial condition:

$$R(k_0) = 1$$

and $S(k_0) = Ent\{0.3 \cdot 1\} = 0$,

$$T(k_0) = 1 - 0 = 1,$$

for the time intervals fixed with the numbers:

$k_{i+1} = k_i + 1$ for $i = 0, 1, 2, 3, \ldots$, and $k_0 = 0$.

Numerical values of the solution of the equations (18) are gathered in Table 1 while the growth function of the process $\int R(.)$ representing the concatenation of two self-replication processes is presented in Fig. 1.

For comparison, in Table 1, in the columns $\int S(k)$ and $\int T(k)$ the values of growth functions of the self-replication processes (16) and (17) has been presented for the independent realization of these processes.

i	$k_i = k$	$\int R(k_i)$	$\int S(k_i)$	$\int T(k_i)$	$\int S(k)$	$\int T(k)$
0	$k_0 = 0$	1	$S(0) + T(0) = 1$		1	1
			0	1		
1	$k_1 = 1$	0+3=3	0	3	2	3
2	$k_2 = 2$	0+9=9	2	7	3	9
3	$k_3 = 3$	3+21=24	7	17	4	27
4	$k_4 = 4$	8+51=59	17	42	5	81
5	$k_5 = 5$	18+126=144	43	101	6	243
6	$k_6 = 6$	44+303=347	104	243	7	729
7	$k_7 = 7$	105+729=834	250	584	8	2187
8	$k_8 = 8$	251+1752=2003	600	1403	9	6561
9	$k_9 = 9$	601+4209=4810	1443	3367	10	19683
10	$k_{10} = 10$	1444+10101= 11545	3463	8082	11	59049
. . .	. . .					

Table 1: Growth function $\int R(k_i)$ of the concatenation of self-replication (16) and (17)

5 CONCATENATION OF THE REPLICATION AND SELF-ORGANIZATION

Let us consider two processes: self-replication with the growth function $\int S(.)$ and self-organization with the growth function $\int U(.)$, described accordingly with the relations:

a) self-replication process:

$$\int S(k_{i+1}) = \int F[\int S(k_i), k_i] \quad (19a)$$

with initial condition $S(k_0) = W_1$, with that:

$k_{i+1} = k_i + A$ for $i = 0, 1, 2, 3, \ldots$, $k_0 = 0$. (19b)

b) self-organization process:

$$\int U(k_{i+1}) = \int G[\int U(k_i), k_i] \quad (20a)$$

with initial condition $U(k_0) = W_1$, with that:

$k_{i+1} = k_i + \int \varphi[\int U(k_i), i]$, $i = 0, 1, 2, 3, \ldots$, (20b)

$k_0 = 0$.

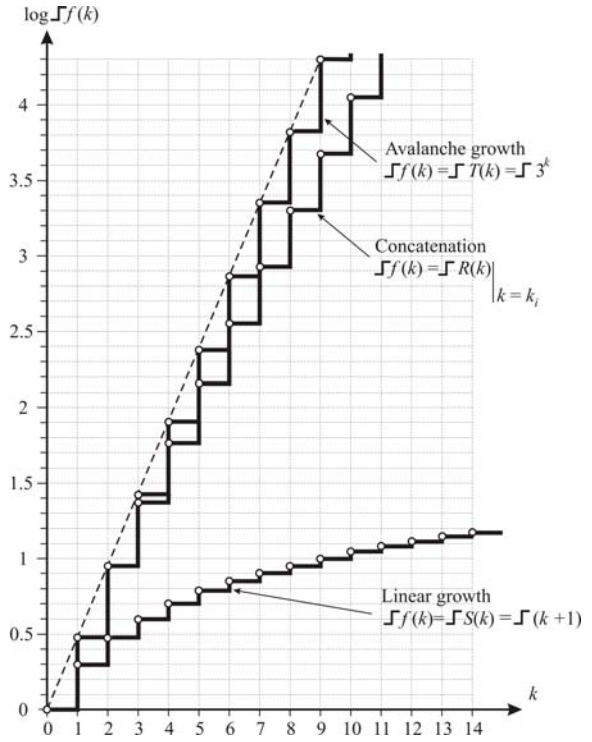

Figure 1: Concatenation of the two processes (α=0.3) of self-replication (16) and (17)

Function $\int\varphi(.)$ enables to modulate the length of the step of self-organization, dependent on the number of free elements existing in the process. Functions $\int F(.)$ and $\int G(.)$, which describe the self-replication and self-organization processes, fulfill the following relations:

$$\int\Delta S = \int S(k_{i+1}) - \int S(k_i) \geq 0, \quad (21a)$$

$$\int\Delta U = \int U(k_{i+1}) - \int U(k_i) \leq 0, \quad (21b)$$

for all *i*.

Therefore, the growth function of the self-replication process is non-decreasing function, on the contrary, the growth function of the self-organization process is non-increasing function (comp. (12a and b)).

6. Concluding Remarks

Considering the concatenation of the replication process (19a) and self-organization process (22a), on basis of Definition 1, we see, that the growth function of the resulting process has different properties than the growth function of the concatenation of two self-replication processes.

In case of the concatenation of *two self-replication processes*, the resulting process is characterized by the growth function which is determining faster growth of elements number than the growth function of the slower self-replication process but slower growth of elements number than the growth function of the faster self-replication process.

In case of the concatenation of *self-replication and self-organization processes*, the resulting process is characterized by the growth function which determines three possible states: the number of elements is growing up, the number of elements is decreasing till to termination of the self-replication process and in a third case the process reaches a stable state i.e. the number of elements arising in the self-replication process is equal to the number of elements snatched in the self-organization process.

The results of the concatenation of self-replication and self-organization processes can be used in the investigation of a connective self-replication of object with the internal molecular program [4], linking of such a processes, and also in analysis of processes where the very complex nanostructures appear e.g. in the processes of molecular biology.

The growing, extinction, and stabilization in the concatenation of self-replication and self-organization processes can be obtained by the mutual choice of the process parameters (19a) and (20a) including also the cycle *A* and modulated cycle $\int\varphi(.)$.

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