



constructed in MATLAB using FEM results. The next step is to predict the system performance under the anticipated disturbance conditions. Section 3.2 outlines the disturbance analysis framework. The disturbance analysis is conducted using the Lyapunov approach. The disturbance analysis results tell us whether the system is able to meet the design requirements or not. If the requirements are met, we may consider to proceed with the current design. If the requirements are not met, then the designer can perform a sensitivity analysis to identify the critical design parameters for improving the system performance. After the critical design parameters are identified, required changes are made and the design process continues until the requirements are met.

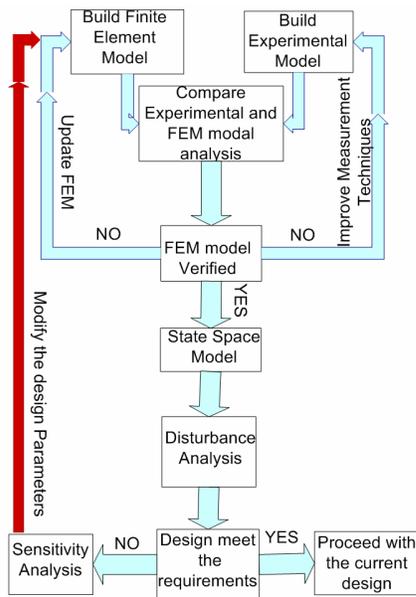


Figure: 2 Design Methodology for MEMS

## 2 FINITE ELEMENT MODELING AND EXPERIMENTAL VALIDATION

### 2.1 Finite Element Model

The Finite Element model of the mirror was built using ANSYS software [6]. The FEM is used to extract the numerical eigenvalues and eigenvectors by conducting modal analysis in the ANSYS environment. The geometric model of the mirror was generated in ANSYS and it was meshed by SOLID 95 elements which can tolerate irregular shapes without as much loss of accuracy. The SOLID 95 element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. However, SOLID95 does not have any rotational degrees of freedom. In this study, the input to our system is the disturbance torque acting on the mirror about the scanning axis. The output or equivalently the performance is defined as the variation of the mirror rotation angle under the effect of a disturbance torque. In order to obtain the relation between the input torque and the rotational

displacement, SHELL 93 elements are attached to the SOLID 95 elements as a thin and low dense layer using the “contact pair” option in ANSYS.

### 2.2 Experimental Model

Advanced testing methods and modal testing systems are necessary for measuring the dynamics of micro systems. In the anticipation of these needs, we built a modal analysis testing system for micro systems in our laboratory. The set-up (see Fig.3) includes a laser doppler vibrometer (LDV) that is used to pick up signals without contacting the micro structure. Details of the set-up and the experimental procedure are described in [7]. The transfer functions that relate the excitation input to velocity output are measured at various locations of the micro structure and then transferred to modal analysis software, ME’scope, to extract the modal parameters.

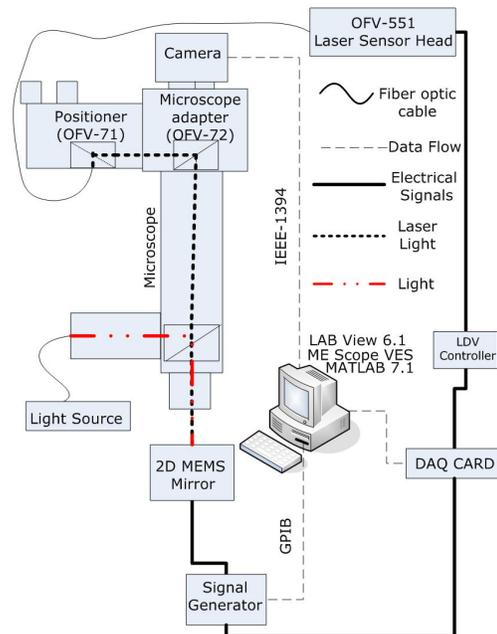


Figure: 3 Experimental Set-up

Experimental model was constructed using the measurement points on the microstructure and the experimental modal parameters are estimated using the curve fitting algorithms built in ME’scope software. The mode shapes and natural frequencies extracted from the measured frequency response functions are given in the second column of Table 1. The third column of Table 1 shows the simulated results obtained by Finite Element Analysis.

The model predicts the dynamic characteristics of the system very well except slight differences in some of the resonance frequencies. The discrepancies between the model and measured results are caused by the inaccuracies in the modeling assumptions and due to the environmental factors such as humidity, dust, and temperature.

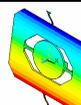
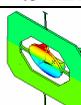
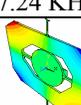
#	Experimental Mode shapes	FEM Mode shapes
1	 0.395 KHz	 0.39 KHz
2	 1.72 KHz	 2.1 KHz
3	 3.83 KHz	 4.5 KHz
4	 21.35 KHz	 21.38 KHz
5	 46.4 KHz	 47.24 KHz
6	 50.8 KHz	 58.9 KHz

Table-1 : Comparison of FEM and Experimental results

### 3 PERFORMANCE PREDICTIONS AND SENSITIVITY ANALYSIS

#### 3.1 State Space Model

First ten mode shapes and frequencies of Ansys modal analysis results are used to construct the state space equations of the mirror. The state space equations representing the system are [5].

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t) \end{aligned} \quad (1)$$

where

$$\mathbf{A} = f(\xi, \omega), \mathbf{B} = f(\Phi), \mathbf{C} = f(\Phi) \text{ and } \mathbf{D} = [0] \quad (2)$$

$\xi$  is the damping factor,  $\omega$  is the natural frequency and  $\Phi$  is the modal transformation matrix obtained from the ANSYS modal analysis results [5]. The modal damping factor  $\xi$  is assumed to be 0.02 and it is consistent with the estimates obtained from the experimental tests [8].

#### 3.2 Disturbance Analysis

After developing the state-space model of the physical system using FEM results, the next step is to predict the performance when the model is subjected to the anticipated disturbances. The performance of optical micro system is directly related to its ability to steer the light beam in the desired direction [5]. In this study, the performance is defined as the rotation of the mirror about the x and y axis. Inputs are defined as torque disturbances acting on the points 2 and 3 (see Fig.1) and outputs are the amount of variance of the rotation at points 1 and 3. This variance should be kept as small as possible in order to improve the performance of the mirror. For stochastic linear systems driven by white noise, the solution of the Lyapunov equation represents the variance of the state vector. The disturbance torque is modeled as the output of a first order shaping filter which is driven by unit intensity white noise [5]. The disturbance filter can be modeled in state-space form.

Placing the state-space form of the disturbance filter in series with the plant equations from equation (1) an overall system of the form can be represented as [5];

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}_{zd}\mathbf{x}(t) + \mathbf{B}_{zd}\mathbf{d}(t) \\ \mathbf{y}(t) &= \mathbf{C}_{zd}\mathbf{x}(t) \end{aligned} \quad (3)$$

Solution of the following steady-state Lyapunov equation leads to the state covariance matrix  $\Sigma_q$ .

$$\mathbf{A}_{zd}\Sigma_q + \Sigma_q\mathbf{A}_{zd}^T + \mathbf{B}_{zd}\mathbf{B}_{zd}^T = 0 \quad (4)$$

This is a matrix equation with the unknown matrix  $\Sigma_q$ , and it can be solved using MATLAB [9]. The performance covariance matrix is given by [5]

$$\Sigma_z = \mathbf{C}_{zd}\Sigma_q\mathbf{C}_{zd}^T \quad (5)$$

The diagonal terms of  $\Sigma_z$  represent the mean-square values and the root-mean-square (RMS) values are simply  $\sigma_{zi}$  [5].

#### 3.3 Sensitivity Analysis

Determining sensitivity of design parameters can provide useful information when the system does not meet the specified requirements. For systems with many design parameters, sensitivity information can identify which parameters in the system are the most significant. These parameters might be the focus of redesign efforts that attempt to improve the performance. In order to compute the sensitivities following expression must be evaluated;

$$\frac{\partial \sigma_{zi}}{\partial p} = \text{Sens. of a performance RMS w.r.t parameter } p \quad (6)$$

The finite difference technique is used calculate the design parameter sensitivities. The design parameters are perturbed one at a time in the finite element analysis, the state space model is reconstructed for the perturbed system, and the performance is calculated using the Lyapunov approach. The performance difference between the perturbed system and the original system is then divided by the perturbed parameter step size and the sensitivity value is approximated. In order to compare sensitivities taken with respect to parameters of different units or magnitudes, the normalized sensitivities are computed as follows.

$$\frac{\sigma_{zi}^{perturbed} - \sigma_{zi}^{original}}{\Delta p} = \frac{\partial \sigma_{zi}}{\partial p} \quad (7)$$

The sensitivity analysis is performed for the 2D scanner for the following design parameters; thickness of slow scan flexure (Tss), width of slow scan flexure (Wss), length of slow scan flexure (Lss), first diameter of the mirror (L1), second diameter of the mirror (L2) and the density of the overall structure (See Fig-1).

	Input 3 vs. output 1 (%)	Input 2 vs. output 3 (%)
Slow Scan Flexure Width (Wss)	-0.59	-0.60
Slow Scan Flexure Length (Lss)	2.16	1.07
Slow Scan Flexure Thickness (Tss)	-1.93	-1.97
Density of the Structure	-0.25	-0.25
1 <sup>st</sup> Diameter of mirror (L1)	-0.61	-0.25
2 <sup>nd</sup> Diameter of mirror (L2)	0.03	-0.33

Table 2- Sensitivity Analysis

Table 2 tabulates the physical parameter sensitivities for the torsional MEMS scanner. Among the computed sensitivities, the greatest sensitivity belongs to the slow scan flexure beam length and thickness. For the case, when we apply an input torque at point 3, 1% increase in the flexure length and thickness will result a 2.16% increase and 1.93% decrease in the RMS performance value of point 1, respectively. A similar behavior is observed when the input torque is applied at point 2 and the output RMS value is measured at point 3 although the percentages vary slightly compared to the previous case. Since the performance is defined as the deviation of the specified point under random disturbances, increasing the beam thickness results in an increase in system stiffness and a decrease in this deviation. When the length of the flexure beam is increased, an adverse effect is expected. Increasing the width of the slow scan flexure results a better performance for both performance output locations. The mirror dimensions are directly related to optical resolution of the system so they are not generally included in the structural redesign efforts. However, the sensitivity results

show that they are as significant as the flexure dimensions in terms of determining the performance of the 2D scanner.

## 4 DISCUSSION AND CONCLUSION

A design methodology for micro systems was presented. A two dimensional micro scanner mirror was chosen in order to demonstrate the developed methodology. The mirror was modeled using finite element modeling techniques to determine the dynamic characteristics. The model predictions were validated utilizing the measured mode shapes and natural frequencies. The model predicts the dynamics of the system very well except slight differences in some of the resonant frequencies. After FEM is validated, the state space model of the system is obtained. The disturbance analysis is performed on the state space model to determine the performance of the system. Sensitivity analysis is performed and the slow scan flexure dimensions are found to be the most significant design parameters for the 2D scanner. Sensitivity analysis is an extremely valuable tool especially when the MEMS have many design parameters. One can easily identify the most significant design parameters and focus on them to improve the performance.

## ACKNOWLEDGEMENTS

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