Simulation and Modeling of a Bridge-type Resonant Beam for a Coriolis True Mass Flow Sensor

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ABSTRACT

This paper presents a simulation of a bridge-type resonating beam connected to the tube loop structure of a Coriolis true mass flow sensor[1]. The resonant beam technique, which is comparable to optical[2], piezoresistive[3], and capacitive[4] detection methods, is used to detect the vertical amplitude change of the tube-loop structure according to the Coriolis force of true mass flow by measuring the frequency shift of the resonating beam. The focus of this paper will be on the frequency shift caused by the amplitude of the bending mode of vibration, and the electrostatic pull-in effect as a measurement method for the frequency shift.

Keywords: resonating beam, Coriolis mass flow sensor, resonant frequency shift

1. INTRODUCTION

It is important to know the true mass flow in many flow applications. The Coriolis mass flow sensors measure the true mass flow. Various methods are used to measure true mass flow, including the optical[2] method, the capacitive[4] method, and the resonant beam detection method which tries to measure the bending or torsion displacement made in Coriolis sensors. Of the above methods, the micromachined resonant beam method is the most sensitive strain sensor. This method is able to provide high quality factor, so the resonant beam is used, not only in Coriolis sensors, but also in pressure sensors and accelerometers.

A resonant beam Coriolis mass flow sensor involves three vibrations. The 1st vibration is actuated externally with the lowest tube torsion resonant frequency depending on flow density. The 2nd vibration, introduced by the Coriolis force is an induced bending mode, and the Coriolis force is given by $F_c = -2mV \times \omega$ where mV is the mass flow momentum and ω is the angular frequency of the 1st torsion vibration. Angular frequency leads to a periodic Coriolis force imposed in the orthogonal direction of the 1st vibration, causing the elastic structure to undergo a 2nd vibration which, orthogonal to the 1st one, is operating in the bending mode of the tube structure. The measurement of mass flow now becomes the measurement of the peak amplitude of the 2nd vibration. The beam is periodically stretched or compressed. The resonant frequency of the beam under this periodic strain will have a periodic shift so that the detection of this frequency shift becomes the measurement of the 2nd vibration amplitude, thus a measurement of the mass flow. This paper presents the simulation of how the resonant beam is used and can be measured by pull-in effect to measure the 2nd vibration amplitude.

2. THEORY

It can be seen how the resonant beam works for measuring the bending displacement using the side view of the beam in figure 2. Assuming the Coriolis force in the z-direction on the top of the tube loop as the displacement, z_c, the bending moment $M(x)$, in the beam at location x is given by[5]

$$M(x) = M - Vx \tag{1}$$

The differential equation of the deflection of the beam is given by [6]

$$\frac{d^2 z_1(x)}{dx^2} = -\frac{M(x)}{EI} \tag{2}$$

Solving the differential equation with the boundary conditions (z(x) =0, z'(x) =0) gives the following [5]:

$$M(x) = \frac{6EIz_c}{L^2}\left(1 - \frac{2x}{L}\right) \tag{3}$$

The x-axis axial tensile stress is given by [5]

$$\sigma_x = c_{11}\varepsilon_x - \nu c_{12}\varepsilon_x \approx \frac{hz_c(c_{11} - \nu c_{12})}{L^2}\left(\frac{6x}{L} - 3\right) \tag{4}$$

where, c_{11} and c_{12} are stiffness coefficients, ν the Poisson's ratio, σ_x the x component of stress, and $\varepsilon_1 = M(x)(h/2)/I$ the x component of strain. The stress is linearly proportional to the Coriolis induced displacement (z_c) of the bending tube loop structure. c_{11} and c_{12} are 16.6($\times 10^{10}$N/m^2) and 6.4, respectively, and ν is 0.27 in single crystalline silicon. The natural frequency of a beam for the first mode is given by [7]

$$f_n = \frac{4.73^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \tag{5}$$

where f_n is a natural frequency of the resonant beam, E is Young's modulus, I bending moment of inertia, ρ density, A cross section of a beam, L length of a beam, and the coefficient for the first mode of natural frequency is 4.73[7].

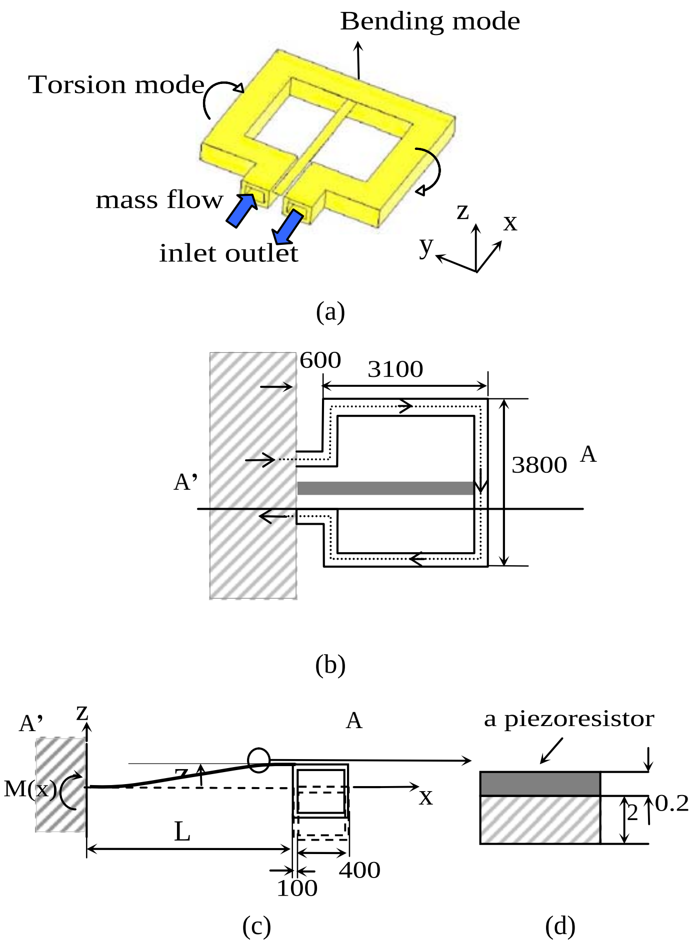
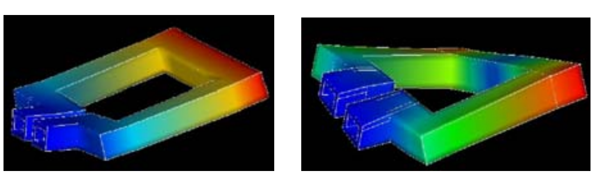


Figure 1: (a) Schematic of a Coriolis mass flow sensor with a resonant beam, (b) Top view of the structure, (c) Side view of the structure across the beam, (d) view of the beam including a piezoresistor. The unit of size is micrometers.

3. SIMULATION

The dimensions of the tube loop structure are shown in figure 1. The natural frequency of the tube loop structure is 2745Hz and 5110Hz in the first (bending) and third (torsion) modes, respectively. We assume the amplitude of the torsion excitation is fully transferred to that of the bending mode after the mass flow passes through the device. The bridge-type resonant beam is first investigated focusing on the relationship among the tensile stress, the resonant frequency shift and the damping effect, which can be controlled.



(a) (b)

Figure 2: the 3D structure of bending mode (a) and torsion mode (b), which are the first and third mechanical vibration modes in the tube loop structure.

The size of a simulated resonant beam is 1000 (length) x 200 (width) x 2 (height) μm^3. Each end of a beam is clamped and one end is displaced along the z-axis from 0 to 1μm. A travel range is considered to be 30 % (i.e. 0.74μm) pull-in displacement of the 2μm gap due to the pull-in effect. The 0.5um displacement for the Coriolis force of the mass flow is applied to avoid the snap down of the beam by pull-in displacement. The tensile stress increases linearly according to the z-direction displacement at the edge of the beam in figure 3. Even if the bending displacement of the tube loop structure is along the z-axis, this displacement is enough to make the x-axis tensile stress significant enough to affect the frequency shift. The frequency shift is nonlinearly proportional to the square of the tensile stress in figure 4. The tensile stress on the top and bottom of the structure may cancel out each other in a beam . However, in this case the beam can be regarded as a kind of a string since the thickness of the beam is 2μm.

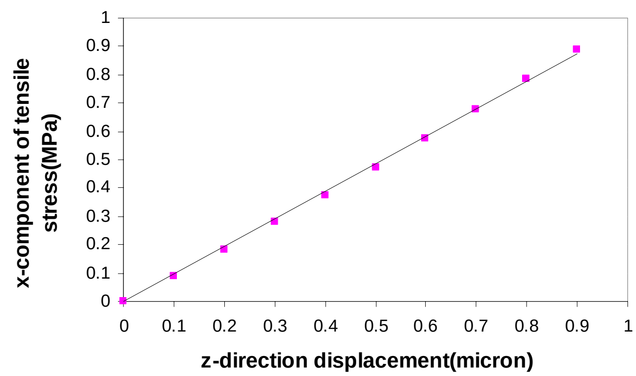


Figure 3: The relationship between the displacement and the tensile stress.

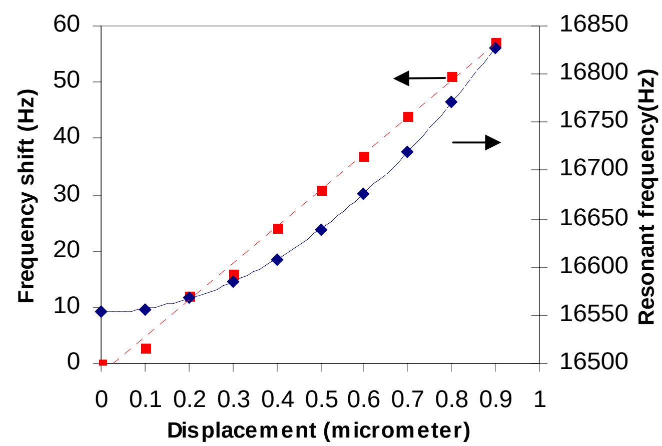


Figure 4: The frequency shift vs. the displacement by mass flow

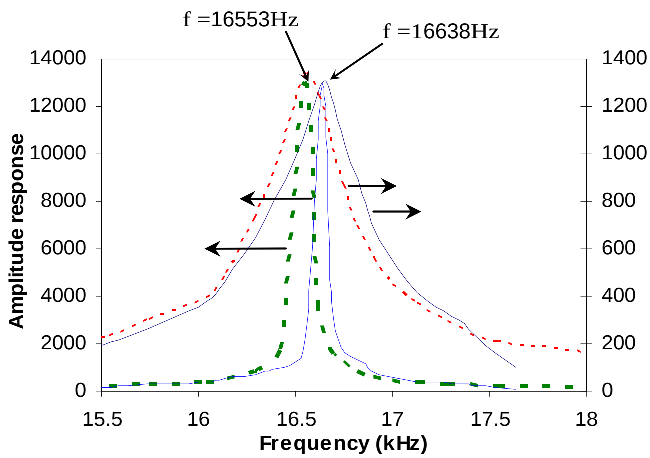


Figure 5: Effect of damping on the resonant frequency; 0.1% (thick lines) and 1% (thin lines) of the critical damping coefficient; dotted lines for no displacement, solid lines for 0.5µm. The resonant frequency shift of 86Hz by 0.5µm displacement is shown by a variation of 0.1% of the critical damping coefficient.

The resonant beam has been simulated with the electrostatic pull-in effect to make the resonance detect the frequency shift. As electrostatic actuation is applied to the resonant beam, the beam is deformed, tensile stress is caused at both ends and the extra frequency shift is produced. The range of the frequency shift caused by the pull-in effect is enough to measure the frequency shift caused by the Coriolis force, as shown in Table 1.

Voltage (V)	Pull-in displacement (µm)	Frequency (Hz)	Capacitance (pF)
0	0	16553	1.052472
2	0.055	16322	1.100262
4	0.245	15574	1.213922
6	0.743	15233	1.306456

Table 1. A comparison of the frequency and the capacitance of the pull-in effect on the resonant beam as the pull-in voltage is applied.

4. DISCUSSION

The resonant frequency shift becomes significant as the resonant frequency increases nonlinearly, in proportion with the square of the tensile stress. However, the ratio of the frequency shift to the tensile stress linearly increases. The electrostatic pull-in effect on the resonant beam has been applied to pick up the frequency shift, instead of using a piezoresistive or a capacitive method. The range of frequency shift produced by the Coriolis force is 86Hz from the 0 to 0.5µm displacement. The range of the frequency shift produced by the pull-in effect is 200Hz and 989Hz at 2V and 4V applied voltage, which means that the sensitivity is 45% and 9%, respectively. The pull-in voltage of 2V applied to the beam facilitates the detection of frequency shift caused by mass flow. The smaller a pull-in voltage on the resonant beam, the greater the sensitivity. There can be a coupling problem between the frequencies of the resonant beam and the tube loop structure, however, the relative higher resonant frequency of the beam compared to that of the tube loop structure minimizes the effects of this problem.

As shown in figure 4 and table 1, the frequency shift increases with tip deflection on the double clamped beam whereas the frequency shift decreases with the pull-in effect. This can be explained by the relationship between the tensile stress and length of a beam. The frequency increases if the increase of length is so small that the tensile stress is the only factor, which affects resonant frequency when tensile force is applied to the axial direction of a beam. Otherwise, the frequency decreases as the length increases. [7] This sensitivity requires a high vacuum system that brings out the small damping coefficient to give a better quality factor, which affects the measurement of the frequency shift. A variation of 0.1% less than the critical damping coefficient is required to sense a significant resonant frequency shift, shown in figure 5.

As noted above, the resonant beam method can be compared with other detection methods for Coriolis mass flow sensors. Optical detection has a high sensitivity, but still needs a larger scale device. The capacitive method can sense a very small amount of capacitance change such as fF, and is comparable to the resonant beam method. The resonant beam method is the most sensitive relative to the other detection methods, but its disadvantage is that it requires very good vacuum sealing for high quality factor. All the above simulations were carried out with the COVERTORWARE software.

5. CONCLUSION

We have simulated the use of a resonant beam structure to measure flow in a Coriolis true mass flow sensor. To illustrate the technique, we have determined a resonant frequency shift (86Hz) for a 0.5µm displacement of the end of a double clamped beam attached to a tube loop structure. We have also studied the pull-in effect as a method to measure the resonant frequency shift induced by mass flow with high sensitivity (45%) at a pull-in voltage of 2V for our test structure.

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