Extending the Validity of Existing Squeezed-Film Damper Models with Elongations of Surface Dimensions

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ABSTRACT

Border flow effects in squeezed-film dampers having a gap separation comparable with the surface dimensions are studied with 2D and 3D FEM simulations and with analytic models derived from the linearized Reynolds equation for small squeeze-numbers. Surface elongations are extracted with 2D FEM simulations for 1D squeezed-film dampers for variable surface topologies for both linear and torsional modes of motion. To model 2D squeezed-film dampers, these elongations are used directly in the compact models, and the results are verified with 3D FEM simulations. FEM simulations show that a simple surface-elongation model gives excellent results, and extend the validity range of existing compact models. To improve the model, drag forces acting on the upper surface and the sidewalls are approximated with simple equations based on FEM simulations.

Keywords: compact model, gas damping, Reynolds equation, squeezed-film damper

1 INTRODUCTION

Compact squeezed-film damper models have been presented for cases where the gap height is very small compared to the surface dimensions [1]–[4]. These idealized models underestimate the damping force, since the open border effects at the surface borders are ignored. Vemuri et al. [5] showed with 3D flow simulations and measurements that the open border effects considerably increase the damping force (35 %), even at surface width / gap height ratios of 20.

In [6] an end effect model was derived based on an assumption for acoustic boundary conditions at the damper borders. FEM simulations presented in this paper will reveal that a simple model with trivial boundary conditions and augmented surface dimensions gives much better results than the model presented in [6].

2 SQUEEZED FILM DAMPER

In the squeezed film problem illustrated in Fig. 1, the force caused by a flat gas film between moving surfaces is modeled with the modified Reynolds equation [2], [7]. Assuming a small pressure change p compared to the ambient pressure P_A, and a small displacement z compared to the static height h of the gap, a linearized form of the modified Reynolds equation results. The linearized equation for isothermal conditions is

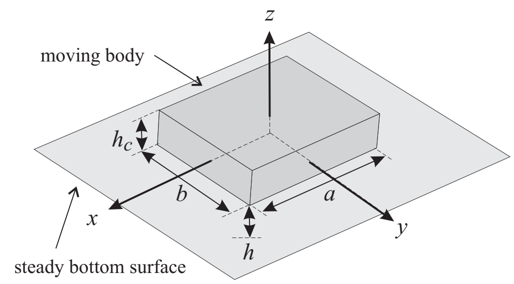

Figure 1: Dimensions of a squeezed film damper. The rectangular body moves causing fluid flow in the air gap below it.

$$\frac{P_\mathrm{A} h^2}{12\eta_\mathrm{eff}}\nabla^2\left(\frac{p}{P_\mathrm{A}}\right)-\frac{\partial}{\partial t}\left(\frac{p}{P_\mathrm{A}}\right)=\frac{v}{h}, \qquad (1)$$

where pressure change p, and the velocity $v = \partial z/\partial t$ are functions of time t and position $(x,\ y)$, and η_eff is the effective viscosity coefficient [2].

An important measure for the squeezed-film effect is the squeeze number $\sigma = 12\eta a^2\omega/(P_\mathrm{A}h^2)$ that specifies the ratio between the spring force due to the gas compressibility, and the force due to the viscous flow. In this paper, only the case $\sigma \ll 1$ is considered.

Traditionally, the squeezed-film problem has been solved applying the trivial boundary condition where the pressure p vanishes at the borders [1], [4], [8]. This condition is justified if the surface dimensions are large compared to the gap height.

However, in practical squeeze-film dampers, the flow escaping from the damper borders might have a significant effect on the damping coefficient. This is true especially when the length and width of the damper are comparable with the air gap height.

2.1 Acoustic Boundary Conditions

The elongation model for a rectangular channel is suggested for calculating the end effect at the borders of a squeezed-film damper in [6]. The border flow can be modeled in a simple way by assuming that the flow channel continues outside the damper borders. The boundary conditions are derived simply from the fact that the pressure changes linearly in this fictitious channel of length $\Delta a/2$:

$$\left.\frac{\partial p}{\partial n}\right|_{\text{border}} = \mp \frac{p}{\Delta a/2}, \tag{2}$$

where n is a coordinate normal to the border.

2.2 Simple Model with Elongations

An alternate model uses solutions calculated with trivial boundary conditions, but applies effective surface dimensions: the length and width of the surface, a and b, are replaced with effective dimensions $a + \Delta a$ and $b + \Delta b$, respectively.

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3.1 Compact Models

The border effects change the behaviour of the viscous flow close to the borders, but their contribution to the gas compressibility is insignificant. This is the reason why the elongation model is derived first ignoring the gas compressibility ($\sigma \ll 1$). The time dependency in the Reynolds equation, Eq. (1), will be ignored, and it is assumed that the width b is infinite and the length a is finite.

For linear motion the velocity is independent of position and we can write $v(x) = v_0$. The trivial boundary conditions are $p(a/2) = p(-a/2) = 0$. This results in a parabolic pressure distribution $p(x)$ and the mechanical resistance $R_{\text{ml},0} = -F/v_0$ is

$$R_{\text{ml},0} = \frac{1}{v_0} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} p(x) \mathrm{d}x \mathrm{d}y = \frac{ba^3 \eta_{\text{eff}}}{h^3}. \tag{3}$$

When applying the acoustic boundary conditions specified in Eq. (2), the mechanical resistance is

$$R_{\text{ml}} = \frac{ba^3 \eta_{\text{eff}}}{h^3} \left(1 + 3\frac{\Delta a}{a}\right). \tag{4}$$

In the torsion motion about the y-axis the velocity of the surface can be written as $v(x) = 2v_0 x/a$, where v_0 is the surface velocity at $x = a/2$. Applying the trivial boundary conditions, the pressure distribution $p(x)$ is solved and the mechanical resistance (force F reduced to $x = a/2$) becomes

$$R_{\text{mt},0} = \frac{2}{av_0} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} xp(x) \mathrm{d}x \mathrm{d}y = \frac{ba^3 \eta_{\text{eff}}}{15h^3}. \tag{5}$$

When applying the acoustic boundary conditions given in Eq. (2) the mechanical resistance becomes

$$R_{\text{mt}} = \frac{ba^3 \eta_{\text{eff}}(a + 6\Delta a)}{15h^3(a + \Delta a)}. \tag{6}$$

3.2 Extraction of Elongations

2D FEM simulations have been carried out to test the accuracy of the end-effect models. The multiphysical simulation software Elmer [9] has been used to solve the incompressible Navier-Stokes equations using stabilized finite element formulation. Isothermal conditions and no-slip boundary conditions are assumed (continuum flow regime). The dimensions of the simulated 1D squeezed-film dampers are shown in Fig. 2 for linear and torsional motion. Generation of the geometry, meshing and FEM simulations were automated with shell scripts. The simulated 2D volume consists of 35000 mesh points.

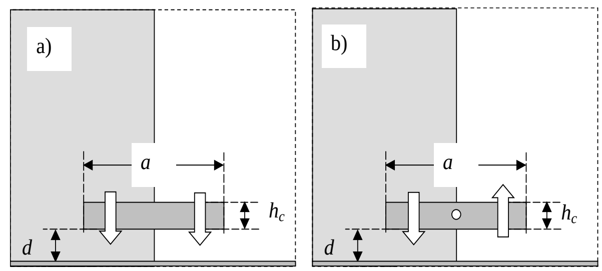

Figure 2: Simulated 2D volumes for a) linear motion and b) torsional motion. The symmetry was exploited to simulate only half of the structure.

A set of simulations was performed, where the dimensions of the structure were varied and the simulated forces (F_{top}, F_{bot}, and F_{side}) or twisting moments (τ_{top}, τ_{bot}, and τ_{side}) acting on all surfaces were recorded. The height of the damper h_{c} was varied from 4 to 32. A nominal surface width b of 1 m was assumed. Flow velocities at the damper surfaces were set to 1 m/s or in case of torsional motion, the boundary conditions (v_{x}, v_{y}) at the damper surfaces were set to imitate the rotation of the damper about the y-axis.

The actual squeezed-film force acts on the bottom of the surface. Forces acting on other surfaces are first excluded from the squeezed-film damper model since they do not obey the typical relation $\approx h^{-3}$. However, these drag forces, that depend also on h_{c}, can be approximated and added to the flow resistance of the damper (see section 3.3).

In the following, the effective elongation is extracted from the simulated force F_{bot} (τ_{bot}) for each ratio a/h. First, a simple elongation model based on the trivial boundary conditions is used: a is replaced with $a + \Delta a_{\text{S}}$ and Δa_{S} is solved from Eq. (3). Next, the elongation based on the acoustic boundary conditions is used. Solving for Δa from Eq. (4) results in the elongation Δa_{A}. Similarly, the elongations for torsion motion are solved from Eqs. (5) and (6). The extracted elongations are shown in Table 1 as a function of a/h.

Table 1: Relative elongations Δa_{S} and Δa_{A} extracted from 2D FEM simulations.

	Linear		Torsional	
a/h	$\Delta a_{\mathrm{S}}/h$	$\Delta a_{\mathrm{A}}/h$	$\Delta a_{\mathrm{S}}/h$	$\Delta a_{\mathrm{A}}/h$
2.0	1.307	2.346	1.654	21.202
4.0	1.307	1.781	1.662	1.009
8.0	1.299	1.522	1.662	0.103
16.0	1.294	1.401	1.650	-0.790
32.0	1.295	1.348	1.640	-2.312

Comparing the extracted elongations Δa_{A} and Δa_{S} shows clearly that the simple elongation model with the trivial boundary conditions is much better than the acoustic resistance boundary condition model. The data shows that the extracted elongations $\Delta a_{\mathrm{S}}/h$ for torsional motion are systematically about 30 % larger than in the linearly moving case.

3.3 Design Equations with Corrections

Based on the presented FEM simulations, novel models are suggested here for 1D squeezed-film dampers where the end effects and drag forces are included. They are given in the form of mechanical resistances for both linear and torsional motion.

For linear movement the average elongation in Table 1 is 1.3. An improved model that accounts for the additional mechanical resistance due to the drag forces $R_{\mathrm{ml,drag}}$ acting on the upper surface and on both of the sidewalls of the damper is $R_{\mathrm{ml}} = R_{\mathrm{ml,elo}} + R_{\mathrm{ml,drag}}$, and

$$R_{\mathrm{ml}} = \frac{b(a+1.3h)^3\eta_{\mathrm{eff}}}{h^3} + 5.0\eta_{\mathrm{eff}} b\sqrt{\frac{a+2.8h_{\mathrm{c}}}{h}}. \qquad (7)$$

Equation (7) is derived heuristically and the coefficients were found by fitting R_{ml} to the simulated total mechanical resistance $R_{\mathrm{ml,FEM}} = (F_{\mathrm{bot}} + F_{\mathrm{top}} + 2F_{\mathrm{side}})/v_{\mathrm{y}}$.

Table 2 shows the importance of the end effect model and the refinement of the model due do the drag force corrections as a function of a/h. If the end effects are ignored, the errors are quite large, from 11 % up to 82.6 %. The elongation model alone is good for $a/h > 8$, but the error of $R_{\mathrm{ml,elo}}$ is as large as 20.4 % for small a/h ratios.

For torsional movement the average extracted elongation in Table 1 is 1.65. According to the simple model with modified dimensions in Eq. (5), the resistance $R_{\mathrm{mt}} = R_{\mathrm{mt,elo}} + R_{\mathrm{mt,drag}}$ is

$$R_{\mathrm{mt}} = \frac{b(a+1.65h)^4\eta_{\mathrm{eff}}}{15ah^3} + 3.2\eta_{\mathrm{eff}} b\sqrt{\frac{a+2.7h_{\mathrm{c}}}{h}}, \qquad (8)$$

where $R_{\mathrm{mt,drag}}$ is an additional term due to the drag forces. R_{mt} has been fitted to the total mechanical resistance $R_{\mathrm{mt,FEM}} = (\tau_{\mathrm{bot}} + \tau_{\mathrm{top}} + 2\tau_{\mathrm{side}})/v_{\mathrm{y}}/(a/2)$ given by the FEM-simulations. Table 2 shows the relative errors of the models for the torsional squeezed-film damper.

Table 2: Maximum relative errors in 1D squeezed-film damper models compared to the FEM simulation results $R_{\mathrm{ml,FEM}}$ and $R_{\mathrm{mt,FEM}}$ ($4 < a/h_{\mathrm{c}} < 32$).

	Linear			Torsional		
a/h	$R_{\mathrm{ml,0}}$	$R_{\mathrm{ml,elo}}$	R_{ml}	$R_{\mathrm{mt,0}}$	$R_{\mathrm{mt,elo}}$	R_{mt}
2.0	-82.6	-20.4	-1.5	-95.4	-49.3	-1.9
4.0	-60.7	-8.7	-0.6	-82.5	-30.3	2.1
8.0	-38.4	-2.5	0.8	-59.6	-14.5	1.5
16.0	-21.6	-0.5	0.7	-35.4	-4.7	-0.8
32.0	-11.4	0.2	0.4	-19.2	-1.2	-0.4

From the large errors in $R_{\mathrm{ml,0}}$ and $R_{\mathrm{mt,0}}$ in Table 2 it can be seen that the contribution of the border flow is considerable in squeezed-film dampers even with moderate a/h ratios. For torsional motion, the border effects are even stronger, for $a/h = 32$ the relative error of $R_{\mathrm{mt,0}}$ is still about 20 %. It can also be seen that in the torsional mode the elongation model $R_{\mathrm{mt,elo}}$ alone is not sufficient for moderate a/h ratios, the drag force correction is necessary for an accurate model.

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4.1 Compact Models

Assuming trivial boundary conditions, the linearized Reynolds equation Eq. (1) can be solved analytically for rectangular surfaces. If the flow in the corners is ignored, the elongation model derived in the previous section can be applied.

For operation at low squeeze numbers the mechanical resistance can be used. For linear motion [2]

$$R_{\mathrm{ml,elo}} = \sum_{m,n} \frac{1}{G_{m,n}(a_{\mathrm{eff}}, b_{\mathrm{eff}})}, \quad \begin{cases} m = 1,3,5,\ldots \\ n = 1,3,5,\ldots \end{cases} \qquad (9)$$

where $a_{\mathrm{eff}} = a + 1.3h$, and $b_{\mathrm{eff}} = b + 1.3h$. For torsional motion

$$R_{\mathrm{mt,elo}} = \sum_{m,n} \frac{1}{G_{m,n}(a_{\mathrm{eff}}, b_{\mathrm{eff}})}, \quad \begin{cases} m = 2,4,6,\ldots \\ n = 1,3,5,\ldots \end{cases} \qquad (10)$$

where $a_{\mathrm{eff}} = a + 1.65h$, and $b_{\mathrm{eff}} = b + 1.3h$ and

$$G_{m,n}(a,b) = \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\frac{m^2n^2\pi^6h^3}{768\eta_{\mathrm{eff}}ab}. \qquad (11)$$

The trivial models $R_{\mathrm{ml,0}}$ and $R_{\mathrm{mt,0}}$ result from $a_{\mathrm{eff}} = a$ and $b_{\mathrm{eff}} = b$.

4.2 FEM Simulations

A set of 3D FEM simulations was performed for a rectangular damper in linear and torsional motion to verify the novel elongation model for 2D dampers. The relative length of the surface a/h was varied from 2 to 32, the relative height of the damper a/h_c was varied from 4 to 32, and a/b was varied from 0.5 to 2. The symmetry of the structure was exploited in the FEM simulations; a quarter of the structure was simulated with 250000 elements for each set of the geometries. The simulations were performed with a 1 GHz Compaq AlphaServer.

4.3 Design Equations with Corrections

To estimate these drag forces, an approximation was used to include the drag force into the mechanical resistances R_{ml} and R_{mt} adding $R_{\mathrm{ml,elo}}$ and $R_{\mathrm{mt,elo}}$ to Eqs. (9) and (10), respectively:

$$R_{\mathrm{ml,drag}} = 2.2\eta_{\mathrm{eff}} a\sqrt{\frac{b+7.1h_c}{h}} + 2.2\eta_{\mathrm{eff}} b\sqrt{\frac{a+7.1h_c}{h}}$$

$$R_{\mathrm{mt,drag}} = 0.72\eta_{\mathrm{eff}} a\frac{0.28a+b+h_c}{h} + 3.2\eta_{\mathrm{eff}} b\sqrt{\frac{a+2.7h_c}{h}}.$$

the resulting relative error is below 6.5 %.

Table 3: Maximum relative errors in 2D squeezed-film damper models compared with the FEM simulation results $R_{\mathrm{ml,FEM}}$ and $R_{\mathrm{mt,FEM}}$.

	Linear			Torsional		
a/h	$R_{\mathrm{ml},0}$	$R_{\mathrm{ml,elo}}$	R_{ml}	$R_{\mathrm{mt},0}$	$R_{\mathrm{mt,elo}}$	R_{mt}
2.0	-90.6	-28.5	1.1	-96.7	-68.6	-4.5
4.0	-72.8	-15.1	-0.7	-86.7	-52.7	4.7
8.0	-47.3	-10.2	1.7	-64.8	-30.3	-2.9
16.0	-26.7	-5.5	6.2	-36.1	-11.0	-3.1
32.0	-11.3	6.0	6.3	-18.9	-2.3	3.0

Table 3 gives the maximum relative errors of various models compared with the simulated total force and twisting moment. The error of the trivial models $R_{\mathrm{ml},0}$ and $R_{\mathrm{mt},0}$, Eqs. (9) and (10) with $a_{\mathrm{eff}} = a$ and $b_{\mathrm{eff}} = b$, is quite large, from 20 % up to 90 %. The elongation model ($R_{\mathrm{ml,elo}}$) with a_{eff} and b_{eff}, is reasonable good for $a/h > 4$. The results for $a/h = 2$ are strongly affected by the drag forces acting on the sidewalls and on the top surface.

The remaining errors in R_{ml} and R_{mt} are larger than in the case of the 1D damper. More accurate drag force approximations could be derived, but it is not justified here, since the accuracy of the 3D FEM simulation is not better than 5% for $a/h \geq 16$. This is due to the insufficient number of elements that could be simulated.

5 CONCLUSIONS

Design aids for squeezed-film dampers were presented in the form of compact models. With the aid of these models, the Reynolds equation has been extended to cases where the air gap size is comparable to the surface dimensions. It was shown that the simple surface-elongation model gives superior results compared to the acoustic elongation model. FEM simulations proved that the drag forces acting on the top surface and on the sidewalls become important, when small a/h-ratios are modeled, especially for torsional motion. The simulations also show clearly that the contribution of the end effects is considerable even at relatively high a/h-ratios.

The elongation model for 1D dampers was usable also in modeling rectangular 2D dampers. Could this approach be extended to general geometry? This would enable the replacement of the 3D Navier-Stokes equation with a 2D Reynolds equation with effective elongations, and the calculation times could be reduced considerably.

This paper considers only the case $\sigma \ll 1$. The extension of the model to higher squeeze numbers is basically straightforward, since the compressibility is not changed by the edge effects. However, in practical topologies where a/h is not small, the inertial [10], acoustic radiation, and rare gas effects become important. Additionally, if rare gas effects are modeled, the length a might be comparable to the mean free path of the gas. In this case the elongation becomes a function of a/h [11].

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