On the correlations between model process parameters in statistical modeling

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1 ABSTRACT

Statistical modeling in the design of today's high performance integrated circuits (IC's) is a necessity to produce competitive products with short development time. The use of backward propagation of variance (BPV) [1,2] has proven its worth among other approaches proposed for the statistical modeling. This methodology introduces physically based **p**rocess and **g**eometry-dependent **p**arameters (PGPs) for each device that is available to the design community in a model library. The goal of this brief contribution is to propose a simple method to establish mathematical relationships among correlated PGPs. The method conforms to syntax-based restrictions that have to be fulfilled in complex model libraries.

Keywords: statistical correlations, statistical modeling, process variations, Monte Carlo simulation

2 INTRODUCTION

Statistical modeling usually denotes a set of tools that enable designers to carry out sensitivity analysis in a particular design, generate case libraries and run Monte Carlo simulations. In the past two decades several methods have been developed to meet these demands. For instance, Principal Component Analysis [3] or Response Surface Modeling [4] can be mentioned. However, large number of measured lots and devices is required. This may be unacceptably time consuming. The methodology exploiting backward propagation of variance [1,2], which is sometimes called Linear Variance Model [5], profits from a sound mathematical background. It takes into account existing correlations between different measured device electrical characteristics *in a single device* (for instance beta, collector and base currents in a bipolar transistor). It guarantees accurate modeling of the measured device characteristics with respect to their mean values and standard deviations. The extra cost for the production of corner lots is avoided. Results can be obtained within one hour on workstations.

The modeling flow is as follows: Let's assume that the following parameters of a particular device have normal distributions described by mean values μ and standard deviations s. The measured $\mu(e_i)$ and $s(e_i)$ of the electrical parameters e_i (often correlated) form an input e-space for *each device* (MOSFETs, BJTs …). They are evaluated from process control (PC) tests. The e-space has a unique relationship to the space of uncorrelated model process parameters (p-space). This relationship between the independent p- and e-spaces is defined by mapping equations [6]. Therefore, it is possible to evaluate means $\mu(p_j)$ and standard deviations $s(p_j)$ of the PGPs. First, note that the resulting sets of $\mu(p_j)$ and $s(p_j)$ are obtained for *each device separately*. Second, for a single device the obtained PGPs are uncorrelated.

3 RESULTS AND DISCUSSION

It is evident that one cannot treat all of the PGPs of all devices as independent. Some of them are related to the same layer, mask or process option. For instance, the epitaxial layer forms the base of substrate PNPs, lateral PNPs and the collector of vertical NPNs as well. Variations in gate oxide thickness affect the n- and p-type MOSFETs and also some capacitors. A mask defining a BJT emitter has an impact on several different types of BJTs. Obviously, it's necessary to introduce a correlation among corresponding process parameters, i.e. among devices that have something in common. A simple transform is proposed. The transform makes sure that correlated PGPs "move in the same direction" and simultaneously it preserves their calculated distributions.

If random variables x_i have normal distributions x_i=N(μ_i,s_i^2) then random variable y

$$y = \left(\frac{x_i - \mu_i}{s_i} \right) \tag{1}$$

has a normalized Gaussian distribution y=N(0,1). This statement is also true visa versa: When a "master" random variable y having y=N(0,1) is introduced, y-dependent variables x_i

$$x_i = y * x_i + \mu_i \tag{2}$$

with *specific* distributions x_i=N(μ_i,s_i^2) can be generated. The effect of the transform is graphically demonstrated in Fig1. Note that non-Gaussian distributions can be also generated applying a suitable transform.

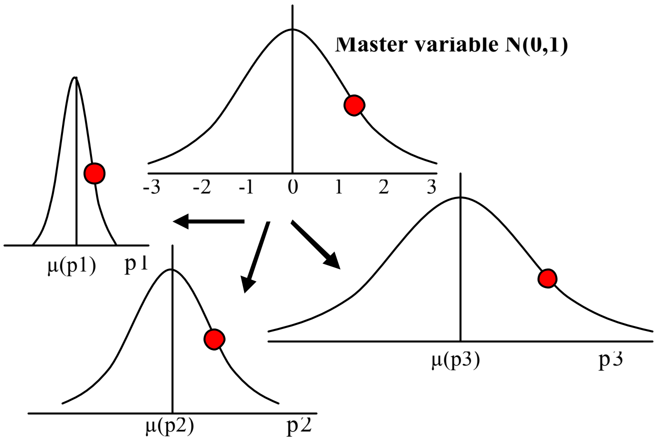


Fig1: Graphical demonstration of the master variable driving a set of correlated p_i (PGPs). Density distributions of p_i are preserved – see the transformed positions of the red point.

As an example we selected a substrate PNPS and a lateral PNPL. First, their base is the epitaxial layer. Therefore, a variation in its concentration has to be reflected in both the SPICE model of PNPS and PNPL. Next, the emitters of both devices and the collector of PNPL are formed by the same mask and implant. It is reasonable to assume that the geometry-dependent PGPs of PNPL and PNPS are also correlated. Again, the correlation is introduced by means of the proposed transform.

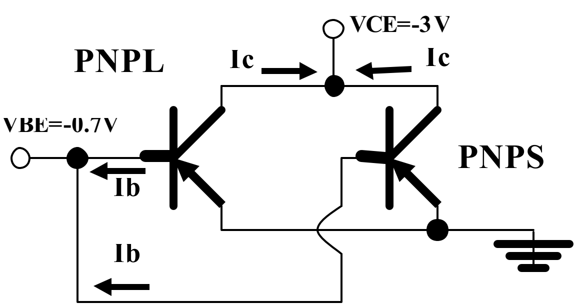


Fig2: Circuit schematics used for Monte Carlo simulations.

An operating point was defined by Vbe and Vce. Monte Carlo simulations were performed (see Fig2 for the circuit schematics). Fig3 shows the results with all the PGPs randomly generated, i.e. uncorrelated. Fig4 presents the outcome with the PGPs appropriately correlated via master variables and the transform described above.

Correlated parameters	Correlation coefficient
Ic_PNPS & Ic_PNPL	0.861
Ib_PNPS & Ib_PNPL	0.940

Table1: Correlation coefficients evaluated from PC data.

The latter graphs in Fig4 are in good agreement with the correlations evaluated from measured PC data in Table1. The message is clear: Fig3 presents combinations that are unlikely to appear. Simulation results with correlated PGPs are much closer to reality.

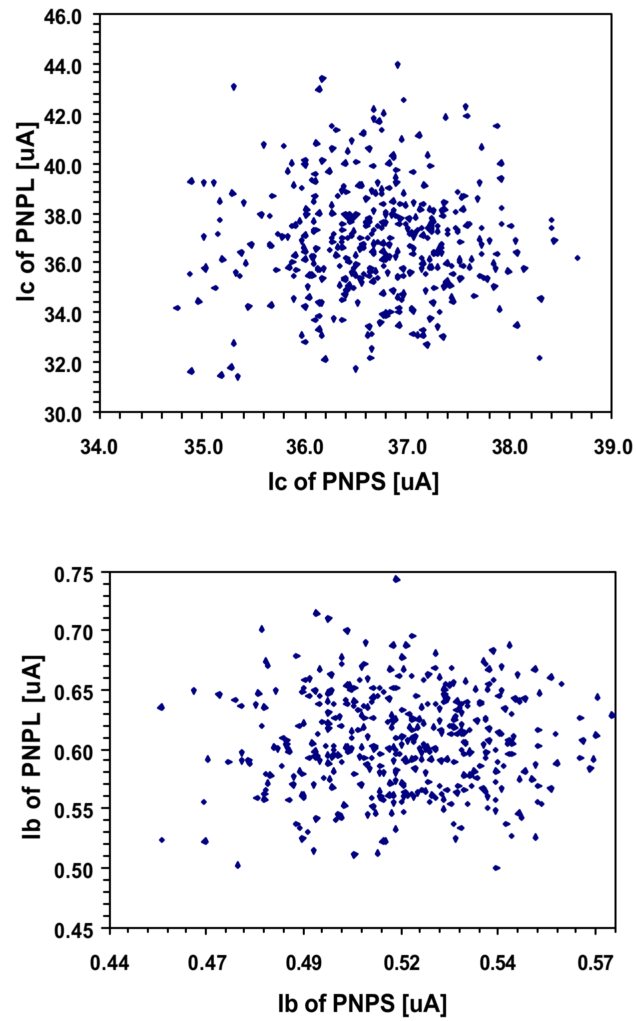
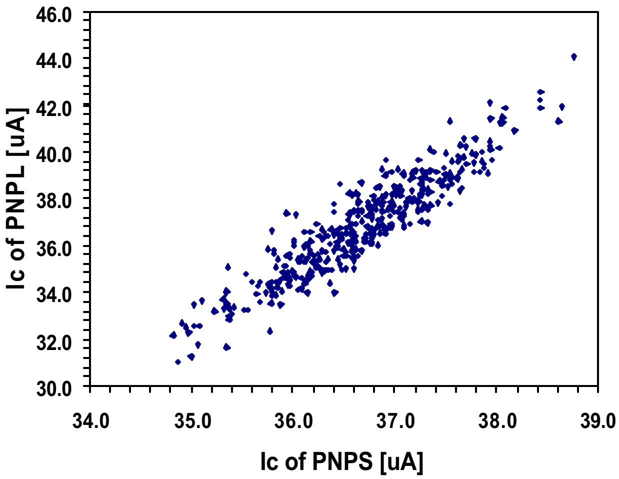


Fig3: Simulated collector and base currents of PNPS and PNPL with **uncorrelated PGPs**, i.e., all PGPs randomly generated according to their calculated distributions (Monte Carlo simulations).

4 CONCLUSIONS

The goal of this contribution was to find a solution to set up correlations among model process parameters. A straightforward transform was suggested. The neglect of correlations in the whole set of PGPs yields excessively pessimistic simulation results. Many of them are unlikely to occur. A new design, that has to accommodate all these situations, would be less competitive due to increased costs and prolonged development time.



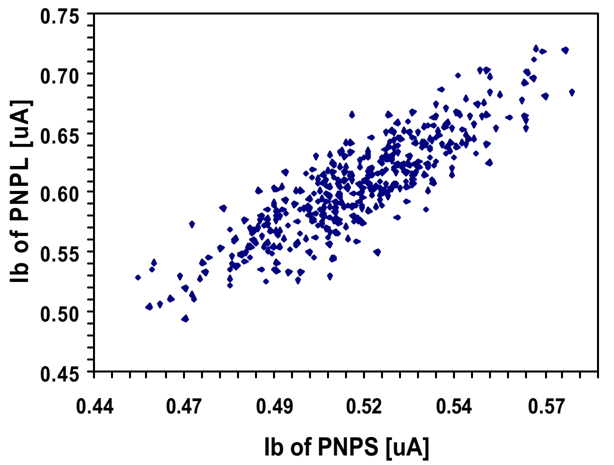


Fig4: Simulated collector and base currents of PNPS and PNPL with **correlated PGPs** via master variables (Monte Carlo simulations).

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