A Systematic Approach to Macromodeling Complex MEMS Devices

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Abstract

This paper describes a systematic approach to lumped-constant modeling of arbitrary MEMS devices based on the concept of element stamps. In particular, it is shown that stamps can be used to represent the dynamical behavior of microelectromechanical elements whose motion is restricted by arbitrary ideal constraints. Since many MEMS structures are built from a common set of basic elements, such as beams, anchors and plates, this approach allows the systematic development of models for a large variety of MEMS components, and their seamless integration into simulators capable of handling indifferently mechanical, electrical or electromechanical devices. As a practical application example, the stamp describing the dynamical model of a planetary electrostatic motor is derived, and numerical results obtained from the simulation of this model are presented.

Keywords: Compact modeling; simulation; MEMS.

1 INTRODUCTION

The combination of MEMS devices with conventional integrated circuit technology can potentially lead to the development of highly efficient, low-cost mixed-technology systems with a wide range of applications. Such systems may contain integrated control circuitry, sensors, digital logic, etc., for a total of hundreds to thousands of tightly-coupled electrical and mechanical devices.

Most of the research work on MEMS simulation has focused on distributed-constant methods of analysis [1]. Recently, however, efforts have been made to develop lumped-constant models for MEMS, which, although less accurate than distributed-constant models, greatly reduce the computational effort required for simulation. Published results indicate that the accuracy of lumped-constant models of MEMS devices is sufficient for many applications [2], [3].

Element stamps were first used to represent lumped-constant device models in conventional circuit simulators [4]. An extension of this modeling approach to the simulation of MEMS has recently been proposed in [5], [6]. Representing MEMS devices by means of stamps has several advantages: it makes it possible to include built-in MEMS device models in simulators, and to handle them no differently than other circuit elements, such as capacitors or transistors. This allows the seamless integration of such models in a simulator capable of handling indifferently mechanical, electrical or electromechanical devices. Furthermore, the use of element stamps greatly reduces the computational effort required for simulation, compared to general-purpose simulators like SABER [2] or MATLAB [3], which rely on user-provided HDL models. This becomes a significant advantage in the simulation of mixed-technology systems containing hundreds or even thousands of electronic and MEMS devices.

This paper describes a systematic approach to the construction of stamp-based models of complex MEMS devices. This approach exploits the fact that many MEMS structures are built from a common set of basic (or atomic) elements, such as beams, anchors and plates, whose motion may be variously restricted by ideal mechanical constraints [7], [8]. It will be shown that the equations describing the dynamics of rigid bodies, as well as any ideal mechanical constraints which they are subject to, can be represented by means of stamps. As mentioned above, stamps can also be used to describe the electrical behavior of a device. Consequently, the electromechanical behavior of a complex device can be modeled by combining systematically the stamps describing the electrical and mechanical behavior of its constitutive elements. As a practical demonstration, this approach will be used to model and simulate the behavior of a planetary electrostatic motor.

2 MEM ELEMENT STAMPS

Throughout this paper it will be assumed that MEM elements can be modeled as rigid bodies, and that their motion is restricted to two-dimensional translations and rotations around an axis perpendicular to the plane of translations. The methodology described herein, however, is of general applicability and remains valid even if these restrictions are removed.

Under these assumptions, the equations of classical mechanics governing the dynamics of rigid bodies can be expressed as follows:

$$\sum_k \mathbf{F}_k = 0; \sum_k T_k = 0, \tag{1}$$

where $\sum \mathbf{F}_k$ is the sum of all the forces acting on the body, and $\sum_k T_k$ is the sum of the force torques with respect to the axis of rotation.

	x_C	y_C	θ	F
mu_x				n_x
mu_y				n_y
$I\omega$			$\frac{\partial T_F}{\partial \theta}$	$\frac{\partial T_F}{\partial F}$
(2)	n_x	n_y	$\frac{\partial g}{\partial \theta}$	

Figure 1: Stamp describing a linear constraint.

Since (1) is linear in $\mathbf{F}_k$ and T_k, each element in the system contributes to it in an easily identifiable way. This means that the entire system of equations can be built by adding the individual contributions of all the basic elements (electrical, mechanical and electromechanical) that make up the system. In this approach, initially developed in the context of conventional circuit simulation, an element's contribution is identified by a so-called *element stamp* or *pattern* [4]. Similarly, MEMS simulation can be performed by developing appropriate stamps for mechanical and electromechanical devices [5], [6].

For example, the forces in the first equation in (1) include the inertial force: $\mathbf{F}_p = -\dot{\mathbf{p}}$, where $\mathbf{p}$ is the body's linear momentum. Similarly, the torques in the second equation include the inertial torque: $T_\pi = -\dot{\pi}$, where π is the body's angular momentum around the axis of rotation. Let (x_C, y_C) be the coordinates of the body's center of mass in a stationary frame of refence, and consider a second reference system fixed with respect to the body, whose origin coincides with the body's center of mass. Let θ identify the rotation of this reference system with respect to the stationary frame of reference. The equations describing the body's dynamical behavior can be written in the following form:

$$\begin{array}{ll} 0 = \dot{x}_C - u_x; & -\dot{p}_x = -m\dot{u}_x \\ 0 = \dot{y}_C - u_y; & -\dot{p}_y = -m\dot{u}_y \\ 0 = \dot{\theta} - \omega; & -\dot{\pi} = -I\dot{\omega} \end{array}$$

where m is the body's mass and I its inertia. Then the body's contribution to (1) consists of the terms on the right-hand side of the equations above. This contribution can be represented by a matrix, or *stamp*, whose entries are obtained by first applying a numerical integration method, such as the trapezoidal method, to the equations, and then linearizing the resulting expression with respect to the device variables [5].

Stamps for more complex elements, whose behavior is described by a set of coupled equations in the electrical and mechanical domains, can be obtained in a similar way [6].

3 IDEAL MECHANICAL CONSTRAINTS

Constraints on the motion of a body introduce additional equations and unknowns into the set of equations describing the system to be simulated, which can be inserted into the element's stamp following a procedure similar to the one described in the previous section. For example, suppose that a body is constrained in such a way that a point P, identified by coordinates (x_P, y_P) in the body's frame of reference, must belong to the stationary line identified by the equation:

$$n_x x + n_y y = c.$$

As the body moves, the coordinates of P in the stationary frame of reference are given by:

$$\begin{aligned} x &= x_C + x_P \cos\theta - y_P \sin\theta \\ y &= y_C + x_P \sin\theta + y_P \cos\theta. \end{aligned}$$

Therefore the constraint that P must belong to the specified line translates into the following equation:

$$\begin{aligned} g(x_C, y_C, \theta) = n_x(x_C + x_P \cos\theta - y_P \sin\theta) + \\ + n_y(y_C + x_P \sin\theta + y_P \cos\theta) - c = 0. \end{aligned} \quad (2)$$

In addition to this equation, the constraint also introduces an additional unknown into the system, namely the the force exerted on the body by the constraint itself. This force is perpendicular to the line, and so can be expressed as $F\mathbf{n}$, where $\mathbf{n} = (n_x, n_y)$ is the vector normal to the line. Its torque with respect to the point (x_C, y_C) is given by:

$$T_F = -F[n_x(x_P \sin\theta + y_P \cos\theta) - n_y(x_P \cos\theta - y_P \sin\theta)].$$

Using the procedure outlined in the previous section, the additional equation and unknown introduced by the constraint can be inserted into the element's stamp, as shown in Fig. 1, where:

$$\begin{aligned} \frac{\partial T_F}{\partial \theta} &= -F[n_x(x_P \cos\theta - y_P \sin\theta) + \\ &\quad n_y(x_P \sin\theta + y_P \cos\theta)] \\ \frac{\partial T_F}{\partial F} &= -n_x(x_P \sin\theta + y_P \cos\theta) + \\ &\quad n_y(x_P \cos\theta - y_P \sin\theta) = \frac{\partial g}{\partial \theta}. \end{aligned}$$

The last line in the stamp corresponds to the constraint given by (2).

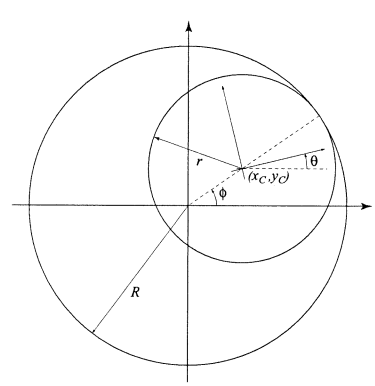

Figure 2: Circular constraint.

	x_C	y_C	u_x	u_y	ω	F_n	F_t
u_x						$x_C/(R-r)$	$-y_C/(R-r)$
u_y						$y_C/(R-r)$	$x_C/(R-r)$
ω							r
(3)	$2x_C$	$2y_C$					
(4)	$u_y/(R-r)$	$-u_x/(R-r)$	$-y_C/(R-r)$	$x_C/(R-r)$	r		

Figure 3: Stamp describing a circular constraint.

As another example, suppose that the body's motion is constrained so that the circle of radius r centered at (x_C, y_C) rolls without slipping inside a bigger, stationary circle of radius R centered at the origin, as shown in Fig. 2. This requirement translates into two mathematical constraints. The first is that the two circles must be tangent to each other, which means that the distance of the center of the smaller circle from the origin must be equal to $R-r$, i.e.:

$$x_C^2 + y_C^2 = (R-r)^2. \tag{3}$$

The second condition is that the velocity of the moving frame at the point of contact between the two circles must be zero, which translates into the following equation:

$$-\dot{x}_C \sin\phi + \dot{y}_C \cos\phi + r\dot{\theta} = 0.$$

But: $\sin\phi = y_C/(R-r), \cos\phi = x_C/(R-r)$, so that the equation above becomes:

$$-\dot{x}_C \frac{y_C}{R-r} + \dot{y}_C \frac{x_C}{R-r} + r\dot{\theta} = 0. \tag{4}$$

Two forces arising from these constraints, F_t and F_n, respectively tangential and normal to the circles at their point of contact, act on the moving body. The components of the sum of these two forces along the x and y axes are:

$$\begin{aligned} F_x &= F_n \cos\phi - F_t \sin\phi \\ F_y &= F_n \sin\phi + F_t \cos\phi, \end{aligned}$$

while the torque with the respect to the point (x_C, y_C) is simply rF_t. The corresponding stamp is shown in Fig. 3.

4 PLANETARY ELECTROSTATIC MOTOR

To demonstrate its practical applications, the methodology described in the previous sections is now used to obtain the stamp describing the dynamical behavior of an electrostatic motor.

A *planetary* or *harmonic* electrostatic motor [9] consists of a single cylindrical rotor placed inside a stator that is divided into a number of electrodes. The rotor is electrically insulated from the stator by one or more layers of dielectric material. The application of appropriate voltages between the rotor and the stator electrodes creates a torque on the rotor that forces it to roll inside the stator.

Using an approximate two-dimensional analysis [9], it can be shown that the capacitance between the i-th stator electrode and the rotor is given by:

$$C_i(\phi_i) = \varepsilon L \frac{1-\alpha}{1-\alpha-\beta} \frac{\Phi(\phi_i + \Delta/2) - \Phi(\phi_i - \Delta/2)}{\log R_0},$$

where ε is the permittivity of the dielectric material between the stator and the rotor, L is the length of the motor, and:

$$\begin{aligned} \alpha &= d/R \\ \beta &= r/R \\ \Phi(\phi) &= \arctan\left[\frac{(a^2-1)\sin\phi}{2a-(a^2+1)\cos\phi}\right], \end{aligned}$$

and a and R_0 are the quantities defined respectively in (2) and (3) in [9]. The total energy stored in the electrostatic field between the stator and the rotor is given by:

$$E = \sum_{i=1}^{N_p} \frac{1}{2} C_i(\phi_i) v_i^2,$$

where v_i is the voltage between the rotor and the i-th stator electrode, and N_p is the number of electrodes in the stator.

The forces acting on the rotor can then be computed from the following equations:

$$\begin{aligned} F_x &= \frac{\partial E}{\partial x_C} = \sum_{i=1}^{N_p} \frac{1}{2} \frac{\partial C_i}{\partial x_C} v_i^2 \\ F_y &= \frac{\partial E}{\partial y_C} = \sum_{i=1}^{N_p} \frac{1}{2} \frac{\partial C_i}{\partial y_C} v_i^2 \end{aligned}$$

(obviously $\frac{\partial E}{\partial \theta} = 0$, so the electrostatic forces do not create any net torque with respect to the rotor's center of mass).

As described in the previous section, two additional equations arise from the constraint that the rotor should rotate inside the stator without slipping. Using the methodology described earlier, these equations can be used to obtain a stamp

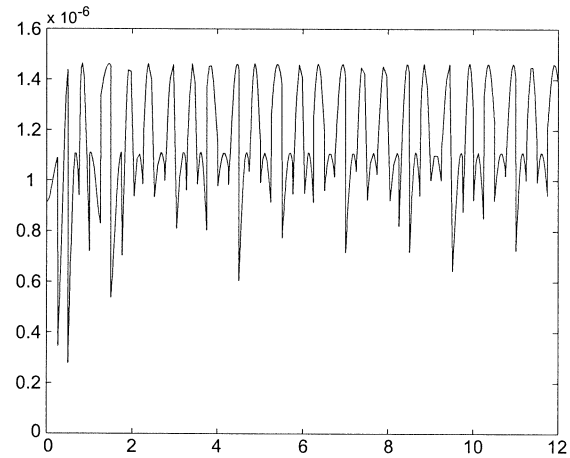

Figure 4: Force on stator, rectangular-wave excitation.

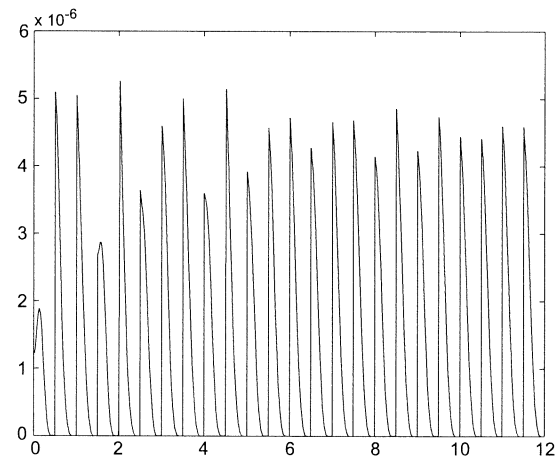

Figure 5: Force on stator, sawtooth-wave excitation.

that completely characterizes the motor's dynamical behavior. The stamp's dimensions are $(N_p+9)\times(N_p+9)$: N_p+1 rows and columns correspond to the stator and rotor voltages, six to x_C,y_C and θ and their derivatives, and two to the constraints imposed on the rotor's motion. The stamp itself is not shown because of space constraints.

5 NUMERICAL RESULTS

To verify the effectiveness of this approach to MEMS simulation, the electrostatic motor stamp derived in the previous section was incorporated into a simulator. A simple system containing a 12-pole motor, with parameters taken from [9], was simulated under various excitation voltages applied to the stator electrodes. Figure 4 shows the normal force exerted on the stator by the rotor when the stator electrodes are driven by periodic 30 V rectangular-wave voltages with a duty cycle equal to 1/8 of their period. By comparison, Fig. 5 shows the normal force acting on the stator when the applied voltages are 60 V sawtooth waveforms with a duty cycle equal to 1/15 of their period. These results give an example of the practical application of the methodology described in this paper to the generation of models suitable for the simulation of complex MEMS devices.

6 CONCLUSION

This paper has described a systematic methodology for the derivation of models of complex MEMS devices based on the concept of element stamp. Stamps provide a compact and efficient way to add the contribution of a particular element to the set of equations describing the overall behavior of a system. It has been shown that arbitrary mechanical constraints can be represented by stamps. Thus dynamical models of complex MEMS devices can be built in a systematic way by combining the stamps corresponding to the device's constitutive elements. A potential limitation of this approach is that a simulator that relies on stamps can handle only devices described by its built-in models. This is unlikely to be a major drawback, however, because most MEMS devices are built from a limited set of elementary elements [8].

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