Resistance measurements and weak localization in long SWNTs

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ABSTRACT

In many IV measurements an evidence has emerged pointing towards the existence of Luttinger liquid behavior in metallic SWNT, as expected for strongly interacting electrons in 1-d, such behavior was observed via the power law temperature and bias-voltage dependence of the current through tunneling contacts attached to the nanotubes. In particular recent advances in the growth of extremely long nanotube (>1 mm) have allowed for experimental measurements on the scaling behavior of resistance in individual, millimeter long SWNT for the temperature range of 1.6-300 K. From the linear scaling of resistance, the temperature dependent electron mean free path has been calculated for each temperature and, beyond the linear scaling regime, it has been observed that the resistance increases exponentially with length, indicating localization behavior. In this work we analyse the results of the resistance measurements of different lengths SWNT indicating the weak localization behavior.

Keywords: quantum transport, carbon nanotube, phase coherence, weak localization

1 INTRODUCTION

The rapidly advancing technology of nanometric devices has led to the production of smaller and smaller systems: among the main items in the design of these electronic devices there are the measurement and understanding of the current-voltage response of electronic circuits in which carbon nanotubes act as conducting elements. These devices, often called mesoscopic systems, are large on the atomic scale but sufficiently small that the electron wavefunction is coherent over the entire sample. The condition for coherence is that the electron traverse the wire without undergoing any inelastic collision: indeed in perfect single-walled carbon nanotube (SWNT) electrons propagate ballistically if the inelastic scattering can be neglected. On the other hand it is also well known that at low-temperature charge transport in any disordered conductor is governed by the interplay between inelastic scattering and elastic scattering off static disorder (impurities and defects) of electrons and, in low dimensions systems like SWNTs, an arbitrarily weak disorder localizes [1,2] all single-electron states and there would be no transport without inelastic processes (Anderson localization[1] of electronic states leads to metal-insulator transition at zero temperature [3]: note that, for electrons in a given conduction band, strong enough disorder can localize the whole band. The metal – insulator transition induced by disorder is called Anderson transition.).

2 QUANTUM TRANSPORT IN SWNT

Coherent quantum transport in low dimensional systems can be investigated with either the Kubo or the Landauer-Buttiker formalism [4]. The first approach, which derives from the fluctuation-dissipation theorem, allows one to evaluate the intrinsic conduction regime within the linear response and gives direct access to the fundamental transport length scales[2] such as the elastic mean free path l_0 and the localization length ξ. While l_0 results from elastic backscattering by static disorder, ξ denotes the scale beyond which quantum conductance decays exponentially with the system length L driving the system from weak to strong localization. The localization length gives the scale beyond which localization effects are fully suppressed owing to mechanisms such as electron-phonon (e-ph) or electron-electron (e-e) coupling: weak localization regime. When l_0 becomes longer than the length of the nanotube between the leads, the carriers propagate ballistically and contact effects prevail. In such a situation, the Landauer-Buttiger formalism becomes more appropriate, since it rigorously treats transmission properties for open systems and arbitrary interface geometries.

2.1 Conductivity and Transport

The more general formula for actual local current measured by experimentalists (generalized Ohm's law) for conductivity in infinite length system is

$$J_\alpha(\mathbf{r},t) = \int d\mathbf{r}' \sum_\beta \sigma_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega) E_\beta(\mathbf{r}',t) \qquad (1)$$

[1] The localization is a property of the states in random QM systems and can be interpreted by total back-reflection of particles from potential barriers so that they become localized in a single potential well.

[2] Important length scales: the coherence length l_φ, the energy relaxation length l, the elastic mean free length l_0, the Fermi wave length λ_F of the electron, the sample size L: in mesoscopic systems it will be $\lambda_F \ll l_0 < L < l_\varphi \ll l$. Note that $k_F l \geq 1$ is called the Ioffe-Regel limit and the atomic Bohr radius $a_0 << \lambda_F$.

where the (non-local) conductivity $\sigma_{\alpha\beta}$ is response to actual (external + induced) electric field and is given by the Kubo formula [5] (where periodic boundary conditions and coupling of only the charge degrees of freedom, none spin degree of freedom is considered, to external $\mathbf{E}$-field are assumed)

$$\sigma_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega) = \frac{e^2}{\hbar\omega}\int_0 dt e^{i\omega t}\left\langle 0\left|[j_\alpha^+(\mathbf{r},t), j_\beta(\mathbf{r}',0)]\right|0\right\rangle + \\ + \frac{ne^2}{m\omega} i\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}') \tag{2}$$

The wave function $|0\rangle$ is the ground state of the many-body Hamiltonian which contains all possible interactions in the solid (except the interaction between the total electric field and the particles of the system) and the first term in (2) is called retarded current-current correlation function. Kubo first derived the equations for electrical conductivity in the solid and Kubo formulas are the name applied to the correlation function which describes the linear response**.**

But this procedure is physically incorrect as a way of defining the conductivity in a finite system in which electrons enter from an external electrode at one end and are removed at the other end collected by another external electrode. In a finite (ideal) sample if the chemical potential is higher at one lead (electrons of the large reservoir with constant μ_1) than at the other lead (with constant μ_2) the current is the response to the gradient of chemical potential for electrons not to the electric field; in other words, if $\mu_1 > \mu_2$ and e is the absolute value of the electronic charge, the voltage difference ΔV between the two baths due to flow across the sample of the current I is

$$\Delta V = \frac{I}{G_c} = \frac{\mu_1 - \mu_2}{e} \tag{3}$$

where

$$G_c = \frac{e^2}{\pi\hbar}\mathcal{T}(E_f) \tag{4}$$

is the irreducible conductance measured between the two outside reservoirs being $\mathcal{T}$ the transmission probability for channel (to go from electrode 1 to electrode 2). The inelastic processes (which break the time-reversal invariance and the phase coherence of the states at the two extremities, dissipate energy and restore equilibrium) in this case are assumed to exist only in the two electrons baths, so that the ramdomized phase of the injected and absorbed electrons through these processes results in no phase relation between particles.

At low temperature, in presence of only elastic scattering for the electrons at the Fermi surface with a linear series of random scatterers connecting the two reservoirs, the true conductance G due to barrier (including spin degeneracy) is correctly given by the transmission and reflection coefficients of the sample by the Landauer-Buttiker , not the Kubo, formula [4,5]

$$G = \frac{e^2}{\pi\hbar}\frac{\mathcal{T}(E_F)}{1-\mathcal{T}(E_F)} \tag{5}$$

Electronic transport measurements [6] on individual SWNT demonstrate that , in the absence of scattering (then the transmission probability is $\mathcal{T}=1$), the momentum relaxation length and the localization length β are much larger than the wire length and the transport in these systems is ballistic: the wavefunction of the electron is extended over the total length of the nanotube and there are only two channels which contribute to the electronic transport giving $G = 2G_c$. However, as already outlined, in presence of some mechanism of scattering the conductance is described by the Landauer formula (4) and the conductance is no longer exactly quantized.

Because the electron can lose energy and equilibrate with heat bath only via inelastic collisions it is necessary to reexamine the conventional concept of energy dissipation in a quasi 1-d resistor systems. In the theory of the 1-*d* electron systems, called Luttinger-liquid (LL)[7,8] (see AppendixA), the correlated electron state is characterized by a parameter *g* that measures the strength of the interaction between the electrons (*g*=1 for non-interacting electrons gas). The most important feature of the LL, in contrast to Landau Fermi-Liquid theory[3] (FL), is the absence of the fermion quasiparticle branch at low energy: excited states of the system must be described by the bosonic fluctuations of the charge and spin densities dispersing with different velocities, which correspond to many-body electron state with a huge number of the electron-hole pairs. This have a pronounced effect on the tunneling into a LL conductor: the IV curve of a tunnel junction between a normal FL and a LL conductor is expected to be non-ohmic and described by a power law with an exponent depending on interaction strength.

Not only, in [9] Bockrath et al. observe a Luttinger-liquid behavior in (rope of) SWNT's in measurements of the electrical transport as a function of temperature resulting in a power laws for linear response conductance

$$G(T) \propto T^{\alpha} \tag{6}$$

where $\alpha = f(g)$ and $g = \left(1 + \frac{2U}{\Delta}\right)^{-1/2}$ (U is the charging energy of the tube and Δ [10] is the single-particle level spacing).

[3] Landau Fermi-Liquid theory is concerned with the properties of many-fermion system at low temperature (much lower than Fermi energy) in the normal state, i.e. in the absence or at least at temperatures above any symmetry breaking phase transition (superconduction, etc).

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Weak localization

When the temperature is so high that the conductivity can be treated as local quantity, like in Anderson localization which deals with wave function of the single electron in presence of impurities, the conductance is obtained by combination of smaller parts of materials. But when the temperature is low (or the sample is small) the dephasing length l_φ is greater than the sample linear dimensions L so that the quantum corrections to the conductivity are non-local and the conductance can no longer be treated as a self-averaging quantity. Since the lack of self-averaging of the conductance is a feature of mesoscopic conductors, in SWNT it is necessary to analyze the effects of weak disorder and the inelastic scatterings on charges transport. Due to the existence of the impurities the transport is more diffusive than ballistic (the existence of scattering is possible in both regimes but in diffusive one we have that $l = v_F \tau << L$ so that the material is characterized by a relatively low mobility) though the elastic scattering of the electrons, if these impurities are equivalent to static defects, can modify the interference terms but does not cause decoherence [4]. At low temperature the conduction take place mainly with electrons at Fermi energy and, due to some gate potential, the Fermi point upon which the electrons travel can be shifted slightly, therefore it is possible that an electron that moved on one side (path) of an impurity begins to move on the other side (path) after the shift. This process (analogous to Aharonov-Bohm effect in the presence of some magnetic field) induced a quantum fluctuation of the conductance of the order of $2e^2/h$ and depends on the exact configuration of scattering centers within the sample: these two paths are time-reversed with respect to one another and since the electron return to its original position it can interfere with itself creating an additional resistance called weak localization [11,12]. Then the weak localization is caused by the quantum interference effect on the diffusive motion of a single electron.

From semiclassical point of view it is possible to calculate this additional resistance considering that the conductivity is related to the current-current correlation function, as in eq. (2), and being $D = v_F^2 \tau$ the diffusion coefficient and $l_\varphi = \sqrt{D\tau_\varphi}$ for 1D we get

$$\Delta\sigma_{WL} \approx \frac{e^2}{\hbar} l_\varphi \left[1 + \frac{\tau_\varphi}{\tau} \right]^{-1/2} \qquad (7)$$

Even though in [13] it has been suggested that zero-point fluctuations cause the dephasing in one dimensional quantum wire at low temperature (ascribed to finite broadening of Fermi surface) and presenting a zero-point-limited dephasing time τ_0 in good agreement with the measured saturation values τ_φ found in many experiments, in [14] this hypothesis is rejected using purely physical arguments.

A more sophisticated theory [15] tells to us that in presence of a vector potential $\mathbf{A}$, for $\tau/\tau_\varphi << 1$, the probability of return path in a disordered SWNT can be conveniently obtained by calculating the Cooperon $C_\varphi(\mathbf{r},\mathbf{r}')$, which is a retarded classical electron-electron propagator satisfying a modified diffusion equation in the frequency domain [16]

$$\left[D\left(-i\nabla_r - \frac{2ie}{\hbar}\mathbf{A} \right)^2 + i\omega + \frac{1}{\tau_\varphi} \right] C_\varphi(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}') \qquad (8)$$

The WL correction to the conductivity is due to enhanced probability to return, so that

$$\Delta\sigma_{WL}(\omega) = -\frac{2e^2 D}{\pi\hbar} C_\varphi(\mathbf{r},\mathbf{r}') \qquad (9)$$

3 RESISTANCE IN LONG SWNT

The conductance of metallic SWNT has been shown to depend strongly on the nature of the contacts between the nanotube and the leads. In a typical experimental setup [17] a bias voltage is applied across a nanotube connected to metallic leads, while a gate voltage applied to a third electrode acts as a chemical potential and modulates the charge on the nanotube (see [17] fgg.1,3 and 4). At room temperature the main origin of the resistivity at low bias in high-quality metallic SWNT is believed to be inelastic scattering by acoustic phonon: the scattering is weak resulting in long mean-free path in a range from few hundred nanometers to several micrometers both in the measurements and in the calculations. Then at low bias regime we get ballistic transport. When sufficiently large bias are applied to drive the electric current, higher energy vibrational modes are activated and e-ph coupling limits ballistic transport: electrons gain enough energy to emit optical or zone-boundary phonons leading to a saturation of the current, in [18] indicated at ~20 μA. The effect of electron-(optical) phonon coupling was found to strongly affect electronic conductance and to induce some energy dependence of the coherence length scale, completely similar to the experimental data obtained in the weak localization regime [19,20].

4 CONCLUSION

We study the suppression of the quasi-ballistic conduction in long (many l_φ) SWNT noting that some environmental conditions also at low temperature introduce some dynamic disorder which involves, by means inelastic scattering [19-21], a weak localization correction to the conductance.

Appendix A LUTTINGER LIQUID: VERY BRIEF REVIEW

A Luttinger liquid (LL) is a one-dimensional (Fermi liquid) correlated electron state characterized by a parameter g that measures the strength of the interaction between electrons: strong repulsive interactions have $g << 1$, whereas $g = 1$ for the non-interacting electron gas (remembering that weakly interacting electrons in normal metal are described by quasiparticles of the Fermi liquid).
The LL's are very special in that they retain a Fermi surface enclosing the same k-space volume as that of free fermions, but there are no fermionic quasi-particles (like in normal Fermi liquids), their elementary excitations are bosonic collective charge and spin fluctuations dispersing with different velocities. An incoming electron decays into such charge and spin excitations which then spatially separate with time (charge-spin separation): the correlations between these excitations are anomalous and show up as interaction-dependent non-universal power laws in many physical properties where those of ordinary metals are characterized by universal (interaction-independent) powers. A list of such properties includes: 1) a continuous momentum distribution function $n(k)$, varying with as $|k - k_F|^{\alpha}$ with an interaction-dependent exponent α, and a pseudogap in the single-particle density of states $\mu |\omega|^{\alpha}$, consequences of the non-existence of fermionic quasi-particles; 2) similar power-law behavior in all correlation functions (in those for charge or spin density wave fluctuations) with universal scaling relations between the different non-universal exponents, which depend only on one effective coupling constant per degree of freedom; 3) finite spin and charge response at small wave vectors and finite Drude weight in the conductivity; 4) spin-charge separation; persistent currents quantized in units of $2k_F$.

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