MICROCHANNEL MIXING, ENTROPY AND MULTIFRACTALS

Miron Kaufman*, Marco Camesasca**, Ica Manas-Zloczower**

*Physics Department, Cleveland State University, Cleveland, OH, USA, m.kaufman@csuohio.edu
**Department of Macromolecular Science, Case Western Reserve University, Cleveland, OH, USA, ixm@case.edu

ABSTRACT

Research aimed at developing rigorous measures of mixing in microchannels is presented. We analyze the experimental data presented by Stroock et al, Science 2002 by using Renyi entropies and multifractal dimensions. Our analysis strongly supports the idea that the microchannel structures are self-similar (fractals).

Keywords: mixing, entropy, multifractals, microchannel

1 INTRODUCTION

Microfluidic systems operate in a pressure driven flow regime with no moving parts to drag the fluids. Mixing can be achieved in such devices by manipulating the geometry of the walls. To design microchannels that are efficient mixers it is important to develop tools for rigorous assessment and mixing quantification. In previous work [1], we proposed to use Shannon and Renyi entropies for assessing mixing efficiency. To further our understanding of mixing, we have also characterized the geometric structures generated by the flow by using multifractal dimensions [2]. In this paper we illustrate the application of entropic and multifractal tools on mixing analysis for the published [3] images of mixing a fluorescent with a non-fluorescent fluid in a microchannel.

2 THEORY: ENTROPY AND MULTIFRACTAL DIMENSIONS

The Shannon [4] entropy S is the rigorous measure of mixing as it is uniquely determined from the Khinchin axioms [5]: it depends on the probability distribution p only; the lowest entropy corresponds to one of the p's being 1, i.e. no mixing; the largest value for the entropy is achieved when all p's are equal to each other, i.e. perfect mixing; S is additive over partitions of the outcomes. For an experiment with M outcomes the Shannon entropy is:

$$S = -\sum_{j=1}^{M} p_j \ln p_j \tag{1}$$

If the last Khinchin axiom is relaxed to consider only statistically independent partitions, Rényi [6] found that the information entropy is replaced by a one-variable function:

$$S(\beta) = \frac{1}{1-\beta} \ln\left(\sum_{j=1}^{M} p_j^{\ \beta} \right) \tag{2}$$

By tuning the parameter β one can focus on different aspects of mixing. For $0 < \beta << 1$ the focus is on empty regions while for $\beta >> 1$ the focus shifts to high density regions. For $\beta = 1$ the Renyi entropy equals the Shannon entropy.

Multifractals [7] are self-similar complex structures generated in natural processes such as diffusion limited aggregation. They are characterized by a spectrum of dimensions $d(\beta)$. The multifractal dimensions are obtained from the linear dependence of the Renyi entropy $S(\beta)$ on $ln(M)$. The slope equals $d(\beta)/D$, where $D = 2$ is the embedding dimension. d(0) is the Hausdorff (fractal) dimension, d(1) is the information (Shannon) dimension and d(2) is the correlation dimension.

3 IMAGE ANALYSIS

We analyze the structures generated in the staggered herringbone mixer and reported by Stroock et al [3] in terms of their multifractal dimensions. The six images are converted to grayscale. Each pixel j has a gray scale value x_j varying between 0, black, and 255, white. It is our working hypothesis that x measures the local concentration of the fluorescent (white) fluid. Renyi entropies are calculated for each image by substituting in Eq. (2) the probabilities computed using the readings from each pixel:

$$p_j = \frac{x_j}{\sum x_i} \tag{3}$$

The dimensions are obtained from a multi-scale analysis of the pictures. For each picture we considered 20 scales of observation starting from 16,058 bins (number of pixels) and going down to 44 bins.

4 RESULTS

In Figure 1 we show the Shannon entropy for the six slides presented in Strook et al [3], measuring the spatial distribution of the fluorescent (white) fluid. We show on the same figure the entropies associated with the negatives of the slides, measuring the spatial distribution of the non-fluorescent (black) fluid. The qualitative conclusion that mixing progresses from section 1 to 6 is confirmed quantitatively by the monotonic increase in entropy.

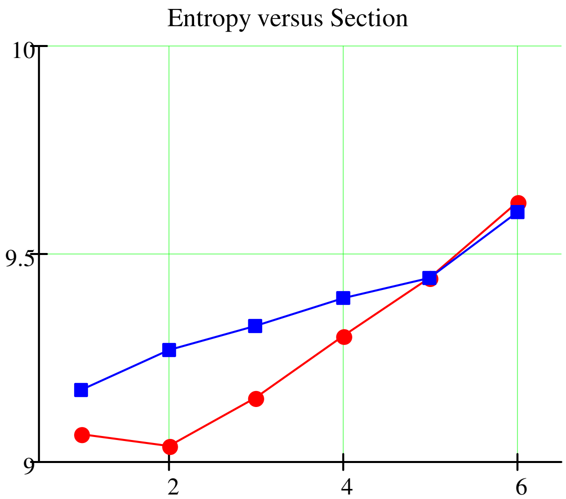

FIGURE 1: Shannon Entropy versus section. Squares – non-fluorescent component; Circles – fluorescent component

Figure 2 illustrates the linear dependence of entropy on the logarithm of the number of bins that is used to estimate the dimensions. The high value of the Pearson correlation coefficient 0.99999 supports the hypothesis that the structure is self-similar (fractal). Very similar results where obtained for all other section and β values.

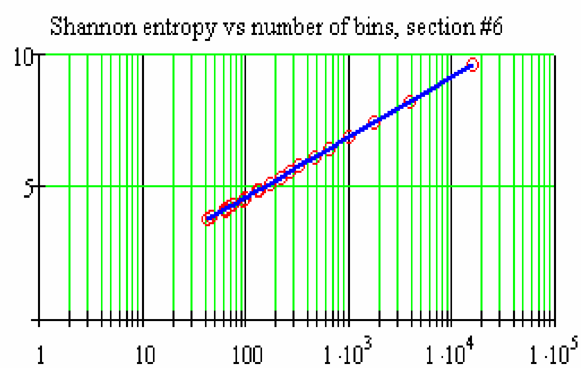

FIGURE 2: Entropy vs. ln(Number bins)

In Figure 3 we show the fractal (Hausdorff), information (Shannon) and correlation dimensions for the six images of Ref.3. The Hausdorff dimension is practically constant at the value of 2. For the other dimensions, the starting section has a dimension close to 2, since the rectangular section is half-filled with one fluid and half with the other fluid. The last section also shows a dimension close to 2, corresponding to a homogeneous configuration. The dimensions are less than two for the intermediate sections, 2 through 5, in view of the less than perfect homogeneity exhibited. The six sections are shown in Figure 4.

FIGURE 3: Hausdorff, information and correlation dimensions vs. section

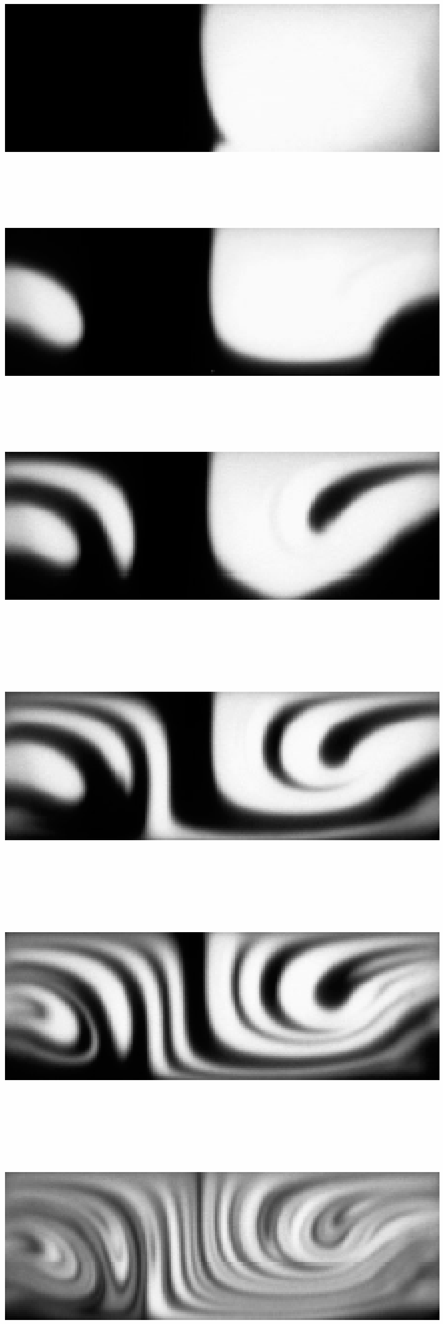

FIGURE 4: Slides 1 through 6 from Ref. 3. We analyze these images using entropic and multifractal measures.

The dependence of the multifractal dimension on section and β is summarized in Figure 5.

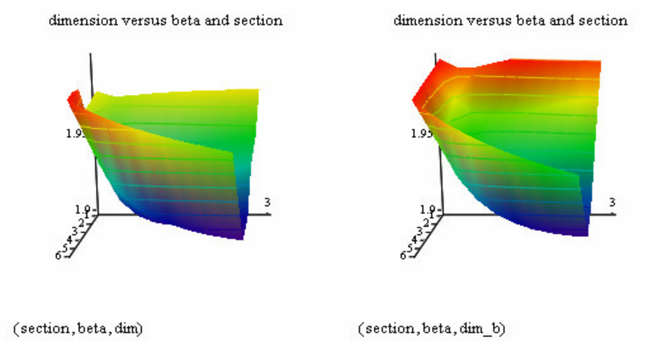

FIGURE 5: Fractal dimensions vs sections and β. Left – fluorescent fluid structure; right – non-fluorescent fluid structure.

5 DISCUSSION

Our direct analysis of the experimental data from the staggered herringbone mixer [3] demonstrates the adaptability of the entropy and multifractal dimensions to both simulations [1] and experiments analyzed via image processing. Future calibration work is needed to study the influence of the type of image file (.bmp, .jpeg, .tif) on the estimated fractal and entropic measures.

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