Solving Electric Field in Combined Conductor and Dielectric Devices

Z.J. Chen*, A. Przekwas*, M. Athavale**, N. Zhou***
* CFD Research Corporation, Huntsville, AL, USA, zjc@cfdrc.com, ajp@cfdrc.com
** GE Power, South Carolina, USA, mahesh.athavale@ge.com
***ESI group, Huntsville, AL, USA, ning.zhou@esi-group-na.com

ABSTRACT

In the design and simulation of micro/nano device areas, it becomes more and more important to obtain, fast and accurately, transient electric field solutions in combined conductor/dielectric materials. The time characteristics for such transient process vary depending on the electric properties of device. Normally, for conductor/dielectric combined device, the transient signal will last in the order of piso second. We have developed a fast, accurate algorithm, based on the conservation of total current, to simulate transient electric field in combined conductor/dielectric device.

Keywords: transient, conductor/dielectric device, total current

1 INTRODUCTION

A typical situation of combined conductor/insulator is RC charging circuit. Figure 1 shows the schematic of an RC circuit.

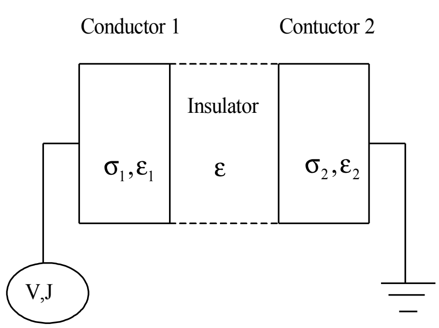


Figure 1: Configuration conductor/insulator of material.

In figure 1, σ and ε are conductivity and permittivity of the media, respectively and V, J are applied electric potential or current, respectively. After the potential or current is applied, electrons accumulate at the interface between conductor and dielectric materials, and build up an electric potential difference across the conductor/dielectric interface. At steady state, the potential difference across the dielectric material equals the applied potential and the electric current in the circuit goes to zero. There are existing two different basic approaches to simulate electric filed: (1) electrostatic approach, which solves the electric field in the dielectric material(insulator) and (2) DC/AC approach, which solves the electric current in the conductor[1]. These two approaches applies current continuity equation into different materials, conductor or insulator, so that a simple form of electric potential equation can be obtained and thus be solved. Once applying the full current continuity equation into the combined conductor/insulator material, the difficulty arises because the charges at material interface have to be dealt. By constructing a "total current conservation" method, we have developed an algorithm to solve electric field in such combined materials.

2 EQUATIONS

The electric current continuity equation is [2]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0 \qquad (1)$$

Where ρ is charge density and J is current density. When using the Gauss' law and introducing the electric field strength, E and electric potential, φ, we have

$$\frac{\partial}{\partial t}(\nabla \cdot \varepsilon \nabla \varphi) + \nabla \cdot (\sigma \nabla \varphi) = 0 \qquad (2)$$

Where, ε is the permittivity and σ is the conductivity of the media. We have used the definition of the electric current density J:

$$\vec{J} = \sigma \vec{E} \qquad (3)$$

We manipulate (2) into the following form:

$$\nabla \cdot (\sigma \nabla \varphi) + \nabla \cdot (\varepsilon \nabla \frac{\partial \varphi}{\partial t}) = 0 \qquad (4)$$

or

$$\nabla \cdot (\sigma \nabla \varphi) + \nabla \cdot \frac{\varepsilon}{\Delta t} \nabla (\varphi^n - \varphi^o) = 0 \qquad (5)$$

The final numerical equation to be solved is[3]

$$\nabla \cdot (\frac{\sigma \Delta t}{\varepsilon_o} + \varepsilon_r) \nabla \varphi^n = \nabla \cdot \varepsilon_r \nabla \varphi^o \qquad (6)$$

Where, ε_0 is the permittivity of vacuum, ε_r is the relative permittivity with $\varepsilon = \varepsilon_0 \varepsilon_r$; n indicates the solution at the new time step and o indicates the solution at the previous time step. Subiterations are performed at each time step to calculate the electric potential for the new time step. Equation (6) is applied in conductor regions. For dielectric materials, since conductivity is zero, electrostatic field equation will be solved

$$\nabla \cdot \varepsilon_o \varepsilon_r \nabla \varphi = \rho \qquad (7)$$

In summary, two equations set will be solved in different regions:

$$\begin{cases} \nabla \cdot (\frac{\sigma \Delta t}{\varepsilon_o} + \varepsilon_r) \nabla \varphi^n = \nabla \cdot \varepsilon_r \nabla \varphi^o, \text{conductor} \\ \nabla \varepsilon_o \varepsilon_r \nabla \varphi = \rho, \quad \text{insulator} \end{cases} \qquad (8)$$

The properties of the materials are in Table 1. The values are in SI unit.

Table 1. Property values for Media

Property	Conductor	Dielectric
Conductivity	1	0
Relative Permittivity	1	1

3 BOUNDARY CONDITION

In the absence of an externally specified surface charge density, the conductor-insulator interface condition is a part of the solution, enforced using the jump condition:

$$\left.\frac{\partial \rho_s}{\partial t}\right|_{\text{int,dielectrics-side}} = \nabla \cdot \sigma \nabla \varphi \big|_{\text{int,conductor-side}} \qquad (9)$$

where ρ_s is the surface charge density at the interface. We can derive a total current from equation (9) across the interface as

$$J^* = (\frac{\sigma \Delta t}{\varepsilon_o} + \varepsilon_r) \nabla \varphi^n - \varepsilon_r \nabla \varphi^o \qquad (10)$$

Then across the interface of conductor/insulator, we have

$$J^* = (\frac{\sigma \Delta t}{\varepsilon_o} + \varepsilon_r) \nabla \varphi^n - \varepsilon_r \nabla \varphi^o = \text{cons tan t} \qquad (11)$$

This gives us the following discritized equation across the interface:

$$(\frac{\sigma \Delta t}{\varepsilon_o} + \varepsilon_r)_c \frac{\varphi_f - \varphi_c}{\delta n_c} = (\frac{\sigma \Delta t}{\varepsilon_o} + \varepsilon_r)_d \frac{\varphi_d - \varphi_f}{\delta n_d} + \left\{ \left(\varepsilon_r E_n^o\right)_d - \left(\varepsilon_r E_n^o\right)_c \right\} \qquad (12)$$

Where, subscripts c, d, and f mean conductor, insulator and interface respectively. δn_c and δn_d are the normal distance from conductor side to interface and insulator side to interface and E_n^o is the normal electric field strength at previous time step. It is obvious that the last two term in eq(12) is the surface charge at the interface at the previous time step. The interface potential can be obtained from equation (12).

After applying Gaussian theorem at the interface, the charge accumulated at the interface is calculated as(reference to figure 1):

$$\begin{cases} \varepsilon \vec{E} - \varepsilon_1 \vec{E}_1 = \rho_{s1} \\ \varepsilon \vec{E} - \varepsilon_2 \vec{E}_2 = \rho_{s2} \end{cases} \qquad (13)$$

Where, E is electric field strength, ρ_s is surface charge density. 1 refers to the interface with conductor 1 and 2 refers to the interface at conductor 2. The above are the basic formulation for the E-field calculations in the combined materials. The formulation includes time-domain calculations of the electric field and currents in the material.

4 VALIDATION CASES

In order to check the accuracy and validity of the algorithm, we simulated several validation test cases. The test cases involved calculation of the transient and steady-state response of different R-C circuits which is combinations of

(1) two conductors and one insulator, and
(2) one conductor and two insulators.

4.1 Two Conductors and one Insulator

Refer to figure 1, two conductors are placed at both left and right ends while the insulator is placed in the middle. we applied a fixed current J = 1 to the right end of conductor and let the left end of conductor to be grounded. This current will build up an electric potential across the insulator. The potential difference across the insulator increases with time due to charge accumulation on the conductor-insulator interface. During the charging process the electric field in the conductor is non-zero, but goes to zero at steady state (i.e. no potential difference across the conductor). We simulated the circuit response in transient mode with a time step of 10^{-11} s. We ran the transient simulation for 200 time steps until the electric field reached a steady state. Presented below are the transient and steady state results. Figure 2 shows the electric potential at different times.

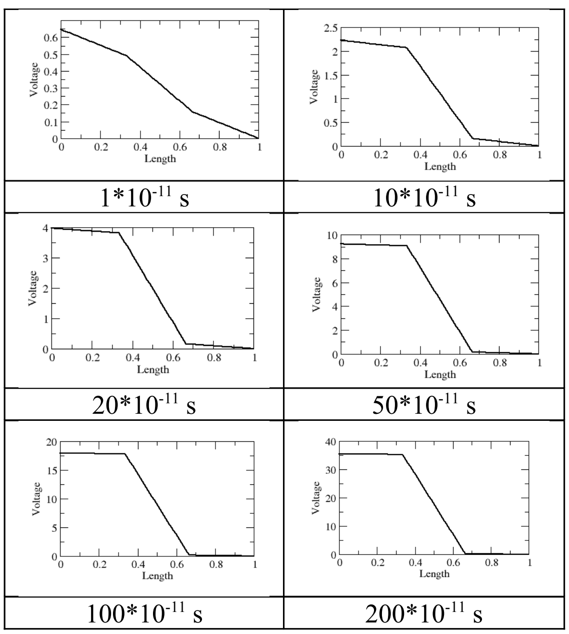


Figure 2: Electric potential distribution along the device at different time.

We can see that, from figure 2, the electric potential across the conductors becomes constant at steady state as expected. During this process, surface charges accumulate at the interfaces between conductor and insulator materials. The charged strength (see fig. 3) increases and reaches a constant value at steady state. Figure 4 shows the distribution of electric field strength along the device. As the applied current builds up the interface charges, the electric field in the insulator increases with time, while the field strength in the conductor approaches zero.

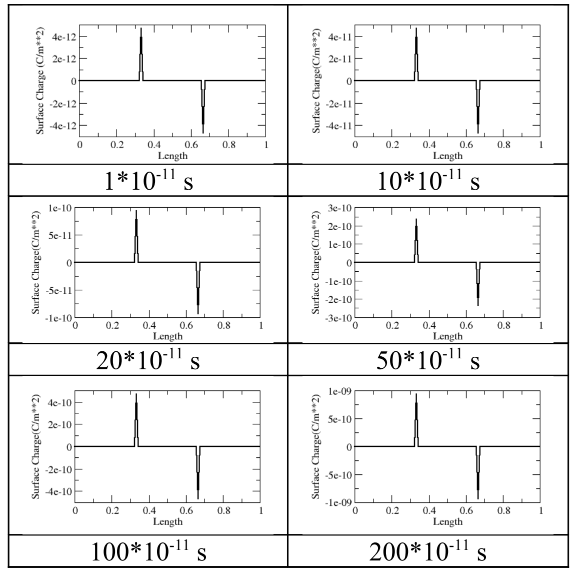


Figure 3: Surface charges at conductor/insulator interfaces at different time.

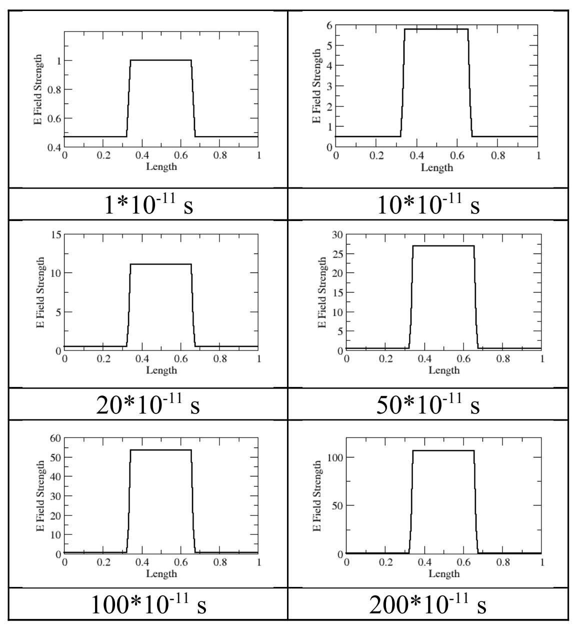


Figure 4: Electric field strength along the device at different time.

This is consistent with Gauss' theory, which indicates that at steady state, an insulated conductor cannot sustain an electric field. From the size of the time steps used in the simulations, we see that the time needed for the electric field in the device to reach a steady state is extremely short.

4.2 Two Insulators and one Conductor

In this case, two insulators are placed at both left and right ends while the conductor is placed in the middle. Since the electrodes are applied on two insulator surfaces, we can only apply a specified voltage as the boundary condition. A potential of 1 Volt was applied at the left end of the insulator with the right end grounded. Figure 5 shows the potential (left column) and electric field strength (right column) distributions along the device. We see that the potential is constant along the conductor at steady state implying zero electric field strength in the conductor. Figure 6 shows the surface charge at the interfaces (left column) and current distribution (right column) along the device. The surface charge approaches a constant level and electric current reaches zero at steady state as expected. In the right column of figure 6, we see the current having an order of 10^{-15} value. This is a machine error.

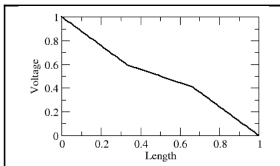
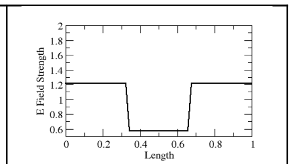
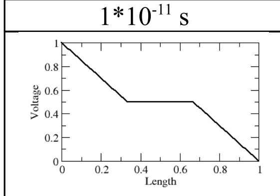
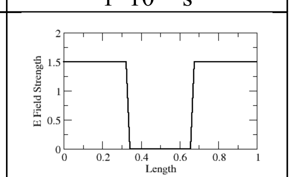
	
$1*10^{-11}$ s	$1*10^{-11}$ s
	
$200*10^{-11}$ s	$200*10^{-11}$ s
Potential	Electric field strength

Figure 5: Potential distribution (left) and electric field strength (right) along the device at different time.

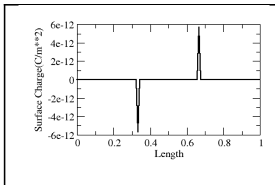
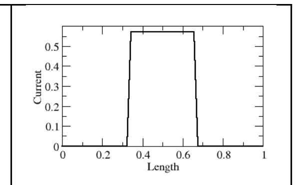
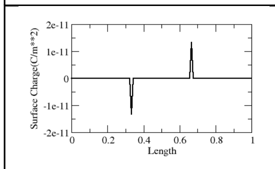
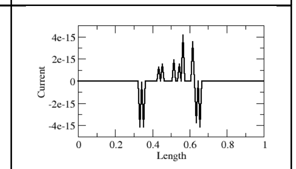
	
$1*10^{-11}$ s	$1*10^{-11}$ s
	
$200*10^{-11}$ s	$200*10^{-11}$ s
Surface charge	Current

Figure 6: Surface charge at interface (left) and current distribution (right) along the device at different time.

5 CONCLUTION

We have shown the “total current conservation” method and applied it to combined conductor/insulator materials to solve electric filed. The solutions show the method is accurate and is able to simulate transient electric filed in combined electric materials.

REFERENCE

[1] CFDRC Technical Report, 1999.

[2] Stratton J.A., Electromagnetic Theory, McGraw-Hill, New York.

[3] Sabelka R., Martins R., Selberherr S.,. Accurate Layout-Based Interconnect Analysis,. Proceedings SISPAD 98 Conf., 1998, Leuven.