Magnetic switch induced electron transport in a double quantum dot.

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1 ABSTRACT

We investigate the possibility of performing single-electron controlled transport in coupled quantum dots based on time-dependent magnetic switches. From solving numerically the time-dependent Schrödinger equation it is shown that certain switches can move an initially localized wave function from one of the dot centers to the other with unit probability.

Keywords: double quantum dots, electron transport, dynamic quantum control

2 INTRODUCTION

Single electron time-dependent quantum control is currently an area of active research. It is motivated by its relevance to operation of solid state quantum computers and in high precision electronic metrology. In both these areas the role of decoherence is an obstacle which can reduce the effect or the accuracy. Therefore, it is of great importance to achieve a sufficient degree of dynamic control of the quantum electronic system [1].

Coupled two-dimensional quantum dots, in particular two quantum wells separated by nanometer distances, are one of the most promising systems for controlled electron transport [2]–[6]. In a recent experiment [7], a single electron initially placed in one quantum well is partially transferred to the other well by exposing the system to a train of short external "top hat" pulses of electric field changes.

An intriguing question is to which extent and under which conditions the transfer can be complete and coherent. This implies that a wave function describing a single unit of charge localized in one of the quantum wells is transferred without dispersion or decoherence to another quantum well. The transfer is induced by some external time-dependent electromagnetic field which can be prepared and turned on/off by external devices.

In this work we explore numerically whether such processes can be controlled by time-dependent external magnetic fields. If the initial state is in a localized eigenstate of the well at a certain value of the external field, a change of the field will induce dynamics. If the dynamics is not blocked by an impermeable barrier the electron can transfer and the challenge is to realize fields which at the right intensity, frequency and duration will cause a complete transfer [8].

The paper is organized as follows: In the next section we describe the numerical model and specify the physical idea behind the switch. In the subsequent section numerical results are presented followed by concluding remarks.

3 THEORETICAL MODEL

Two coupled quantum wells are conveniently described by a double harmonic oscillator potential [**?**,**?**] given by

$$V(x,y;d)=\frac{1}{2}m^*\omega_0^2\min\left[\left(x-\frac{d}{2}\right)^2+y^2,\left(x+\frac{d}{2}\right)^2+y^2\right]. \quad (1)$$

The inter-dot distance which separates the two wells is d and ω_0 is the confining trap frequency [10]. The single-electron Hamiltonian reads,

$$H(x,y,t)=-\frac{\hbar^2}{2m^*}\left(\partial_x^2+\partial_y^2\right)+V(x,y;d)+V_{ext}(x,y,t), \quad (2)$$

where $V_{ext}(x,y,t)$ is the external time- and field-dependent potential,

$$V_{ext}(x,y,t)=\frac{1}{2}w_B^2(x^2+y^2)+w_BL_z. \quad (3)$$

The external potential thus describes a magnetic field parallel with the $\hat{z}$-direction. The spatial extent of this direction is assumed to be so small that this degree of freedom can otherwise be considered frozen. Then, $\omega_B=\frac{eB(t)}{2m^*}$ is the time-dependent Larmor frequency and $L_z=i\hbar y\partial_x-i\hbar x\partial_y$ is the angular momentum operator.

The dynamics is governed by the two-dimensional time-dependent Schrö dinger equation,

$$i\hbar\partial_t\Psi(x,y,t)=H(x,y,t)\Psi(x,y,t), \quad (4)$$

, to which a numerical solution can be obtained by expanding the wave function in a basis set containing a large number of eigenstates of the $d=0$ a.u. harmonic oscillator in Cartesian coordinates,

$$\Psi(x,y,t)=\sum_{n,m}a_{nm}(t)\phi_{n,m}(x,y) \quad (5)$$

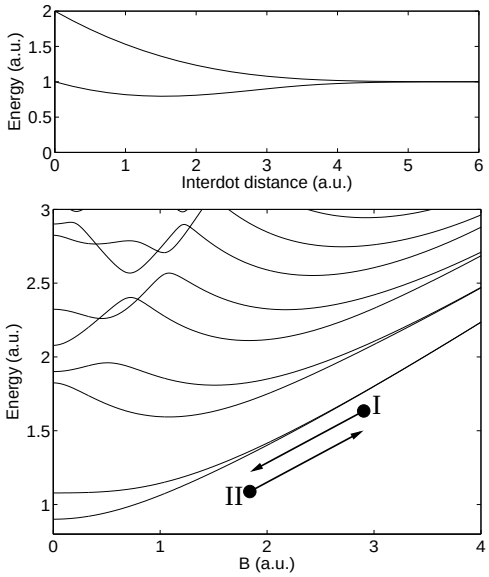

Figure 1: Upper: The lowermost energy levels of the double quantum dot as a function of inter-dot distance d ($\omega = 1$). Lower: Energy levels of the double dot system with $d = 3$ a.u. (100 nm) as a function of the magnetic field strength. The magnetic field points in the $\hat{z}$-direction and the transition arrows indicate a field change process.

which by standard projection techniques leads to a set of coupled differential equations for the amplitude vector $\mathbf{a}(t) = (a_{00}, a_{01}, a_{10}...)$,

$$i\hbar\partial_t \mathbf{a}(t) = \mathbf{H}(t)\mathbf{a}(t). \quad (6)$$

This set of equations is then integrated with the appropriate initial conditions by an efficient and stable numerical method [11]. Reduced atomic units (a.u.) are applied in the actual calculations, defined by Planck's constant, the effective electron mass and the unit charge [10].

For sufficiently long inter-dot distance d the two wells couple so weakly that local states in either well are stable eigenstates. The energy spectrum in this limit is very close to the spectrum of independent Harmonic oscillators, at least for a large number of low laying states. This is also true for the limit $d \to 0$. In the intermediate region these degenerate energy levels are splitting. If the inter-dot distance could be varied with time, similar to the internuclear distance in diatomic molecules, the variation of d would lead to energy splitting and induce dynamics similar to gerade-ungerade charge cloud fluctuations in a diatomic molecule [9]. A sudden increase of d at a certain time would then have lead to complete charge transfer. This idea is illustrated in the upper panel of Fig. 1, showing the lowest energy levels followed by a diagonalization of Eq.(2).

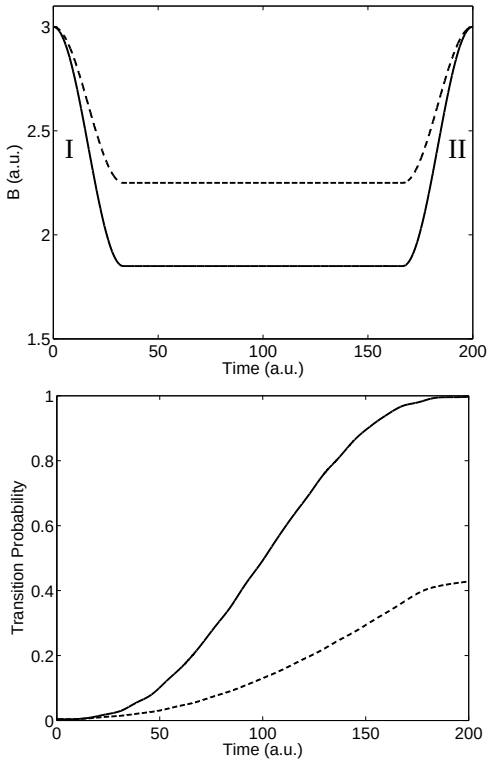

Figure 2: Upper panel: Variation of the magnetic field with time. The change of the field at I and II corresponds to the transition arrows indicated in Fig. 1. Lower panel: The probability of finding the electron in the state $|R\rangle$ versus time. Solid line: a -case where a complete charge transfer is achieved, dashed line: a partial transfer.

Unfortunately, present technology does not allow a fast variation of the inter-dot distance. Alternatively the effective inter-dot distance can be manipulated by electromagnetic fields. In the lower panel of Fig. 1 we display the energy curves of Eq.(2) in the presence of a magnetic field. A number of crossings and avoided crossings is seen. The splitting between the two lowest energy levels is seen to decrease with increasing field strength. Thus the magnetic field strength can play the same role as a virtual variation of the inter-dot distance: A localized initial electron in a strong magnetic field will be allowed to couple to the other quantum dot if the field strength is suddenly reduced. In the next section we explore this possibility.

4 RESULTS

In the following calculations we use $\omega_0 = 1.0$ a.u. and the magnetic field strength is varied with time as shown in the upper panel of Fig. 2.

As described in the previous section, we start with an electron localized in the left well, its initial state is a lin-

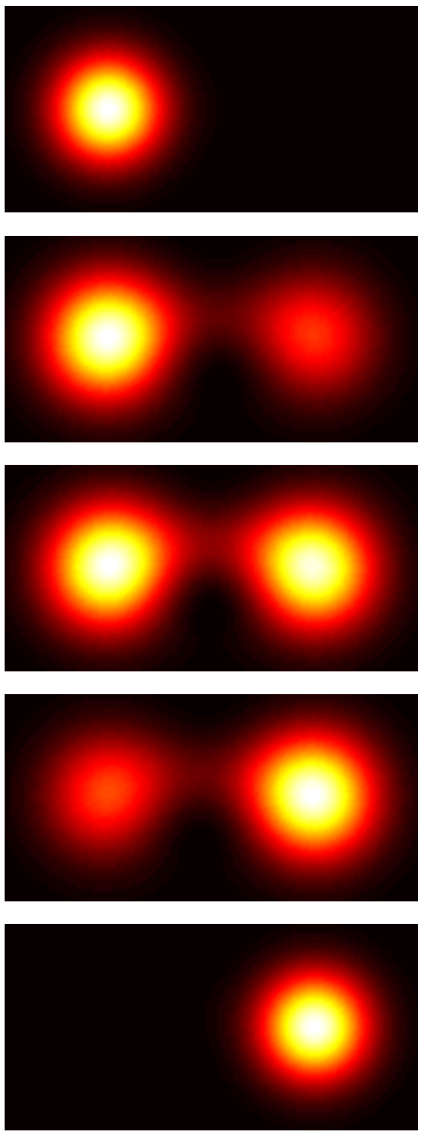

Figure 3: (color online). Snapshots of the electronic probability density for five different stages of the magnetic switch, $t = 0$, $t = 80$, $t = 100$, $t = 120$, and $t = 200$ a.u. ($\sim$ 132 ps). The axis ranges are from $x = -3$ to $x = 3$ a.u. (horizontal direction) and $y = -1.5$ to $y = 1.5$ a.u. (vertical direction).

ear combination of a *gerade* and an *ungerade* molecular state, $\Psi_L = \frac{1}{\sqrt{2}}(\psi_{sym} + \psi_{antisym})$. The magnetic field is then suddenly switched down to around $B = 1.8$ a.u, which decreases the effective inter-dot distance. The charge cloud then start to tunnel between the dot centers. After $t \sim 132$ ps the magnetic field is turned on again to $B = 3$ a.u., and the electron is settled in the "right" state, $\Psi_R = \frac{1}{\sqrt{2}}(\psi_{sym} - \psi_{antisym})$.

The upper panel of Fig. 2 shows how the magnetic field strength changes with time, corresponding to field change in I and II in Fig. 1. The lower panel shows the probability of finding the electron in the lowest local state of the right well versus time. The transition is seen to be complete at 200 a.u, or $t \sim 132$ ps. We notice that the strength of the magnetic field is of crucial importance to achieve a complete electron transfer. A choice of a too strong magnetic field (upper panel of Fig. 2, dashed line) results in only a partial transfer (around 40%), as seen in the lower panel, dashed curve.

In Fig. 3 we show in more detail how the electronic probability density changes in time. In the first snapshot the electron is in the initial localized state in the left well. In the next three snapshots the charge cloud is gradually transferred from the left to right well through delocalized states. The slight asymmetry in y-direction observed in the middle panels arise from our choice of the ground state wave function. The last snapshot shows the electron as it has become fully transferred to the right-state and by inspection a complete (more than 99.9%) transition is achieved.

5 CONCLUSIONS

We have demonstrated a principle behind a fast magnetic switch that may transfer an electron between the two wells of a double quantum dot with 100% probability, avoiding the problem of quantum dispersion. We conclude that such a transition can be achieved with an appropriate setup of the time-dependent magnetic field. The transition is performed at a time-scale of a hundred picoseconds (132 ps) and with realizable magnetic field strengths. These properties of the double dot system can make it a suitable candidate for a logical gate with possible applications in quantum computation.

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