Technique for Time and Frequency Dependent Solutions of the Diffusion Equation: Application to Temperature of Nanoscale Devices

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ABSTRACT

An equivalent circuit technique is presented that yields frequency and time dependent solutions of the diffusion equation. Circuit simulations can then be used to describe time dependent temperature variations of nanoscale devices. The technique is applied to describe frequency and time dependent variations in the temperature of nanoscale electron devices.

Keywords: diffusion, heat flow, nanoscale devices

1 INTRODUCTION

Typical CMOS device currents have remained around 1mA/µm while devices have scaled down, in the 2000 time frame 0.1µm, 100nm, devices used 1.5 V power supplies. The projection into the 2010 time frame is 0.01µm or 10nm dimensions with 0.5V power supplies which results for a minimum dimension device in power densities of $5x10^6$ W/cm^2. Such power densities will result in very high device temperatures even with ideal heat sinks. Not only is there the problem of high steady state device temperatures but fluctuations and variations about these high device temperatures are to be expected, these variations and fluctuations become much larger with nanoscale devices

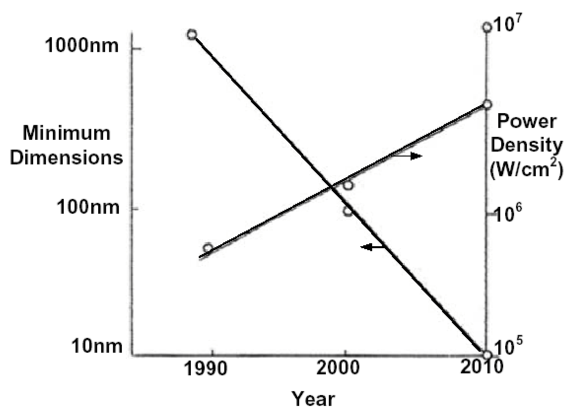

Figure 1: Increase in power density in integrated circuits with time and decreasing feature sizes.

2 HEAT FLOW

A textbook formulation of the problem of frequency dependent solutions to the diffusion equation has been given by Kittel and Kroemer[1], where they give no solution but note that high frequency variations will be strongly attenuated.

What is new here is our treatment of time and frequency dependent solutions to the diffusion equation by transmission line techniques and the application to nanoscale devices[2,3]. The diffusion equation in rectangular coordinates describing heat conduction [1] is;

$$a \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \tag{1}$$

where a is the thermal diffusivity, m^2/sec in MKS units and T is the temperature.

This is the same type of equation as that describing voltages on a distributed R-C transmission line in electric circuits.

$$\frac{\partial^2 V}{\partial x^2} = R\ C\ \frac{\partial V}{\partial t} \tag{2}$$

These R-C transmission lines have been previously analyzed [2, 3] and are diffusion lines where potential and currents are described by the diffusion equation. An equivalent circuit representation can be made in rectangular coordinates for heat conduction as shown in Fig. 2 where for each volume element,

$$R = 1/\mathrm{K}A \qquad K/W \cdot m \tag{3}$$

$$C = C_p \rho A \qquad J/K \cdot m \tag{4}$$

where K is the thermal conductivity, C_p the heat capacity, ρ the density, and A the area of the sample through which there is heat conduction. Temperature is analogous to voltage and heat flux analogous to current. The time invariant steady state, or DC, solution for this line is then a linear variation in temperature where the heat flux; $flux = \mathrm{K}\ A\ dT / dx$ and for the total line $flux = \mathrm{K}\ A\ \Delta T / d$, and $R_{DC} = d / (\mathrm{K}\ A)$. Thermal conductivity and diffusivity

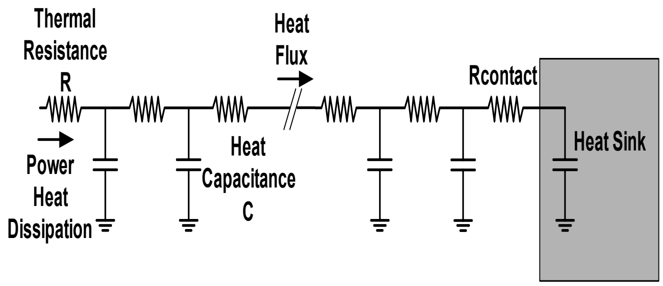

Figure 2: Equivalent circuit for heat flow.

are not independent but are related in a form which we will find useful,

$$\frac{a}{\mathrm{K}}=\frac{1}{C_p\,\rho} \qquad \frac{K\cdot m^3}{J} \tag{5}$$

The steady state time dependent solutions of this differential equation and transmission line of length l=d, in response to a sinusoidal excitation in temperature T at the sending end of this line are described by the AC impedance looking into this line, Zs.

At very low frequencies, however, the sending end impedance is just $Z_s = Z_o \tanh\gamma l \approx Z_o \gamma l$ but this is just R_{DC}. At high frequencies this line is strongly attenuating [1] and the impedance looking into this long lossy line is just Zo, but $Z_o = R_{DC} / \gamma\, l$ where $\gamma = \sqrt{Z\,Y} = \sqrt{R\, j\omega\, C}$ but RC = 1/a and then $\gamma = \sqrt{j\,\omega / a}$.

Thus we can find simple solutions for the two limiting cases, DC and high AC frequencies.

$$Z_s\ (DC) = R_{DC} \tag{6}$$

$$Z_s\ (High\ Frequency\ AC) = Z_o = \frac{R_{DC}}{\gamma l} \tag{7}$$

where

$$\gamma\ l = \sqrt{j\,\omega\,\frac{l^2}{a}} \tag{8}$$

and then $|Z_s|^2 = R_{DC}^2 / |\gamma\ l|^2$ where $|\gamma\ l|^2 = \omega\ l^2 / a$. If we now let $\omega_{cth} = a / l^2$, which is one half of $2\ a / l^2$ or the reciprocal of the diffusion transit time, then,

$$|Z_s|^2 = \frac{R_{DC}^2\, a}{l^2\,\omega} = \frac{R_{DC}^2\,\omega_{cth}}{\omega} \tag{9}$$

3 CIRCUIT SIMULATIONS

Well known circuit simulation programs such as generic SPICE or commercial adaptations of this program, as PSPICE, can be used to determine the frequency response or the time response of excitations to the R-C diffusion line. Resistance and capacitive circuit elements are calculated for each volume element, dx, in rectangular coordinates or, dr, in spherical or cylindrical coordinates. A current source, representing power dissipation or heat flux at the input of the line is applied, and the temperature as represented by voltage at the sending end of the R-C diffusion line calculated. This excitation can either be frequency dependent, for instance representing AC power dissipation in a nanoscale device, or time dependent representing for instance the sudden initiation of a steady state power dissipation in a nanoscale device, or a pulse of power causing a sudden increase in temperature of the volume element at the input of the line.

As an example we consider silicon on insulator technology and the heating of the small area of a silicon transistor on an oxide layer by a power pulse. The maximum temperature excursion of the small volume, Fig. 3(a), in which the energy of the power dissipated is absorbed is determined by the heat capacity of this volume which is semiconductor, in this example silicon. In the rectangular coordinates, an equivalent circuit representation can be made for the heat capacity as shown in Fig.3 (a) where for the volume element,

$$C = C_p \rho\, s^2 t \qquad J/K \tag{10}$$

where C_p the heat capacity, ρ the density, and, s^2, the surface area of the heated portion of the semiconductor layer of thickness, t. For the case of a power pulse the temperature change will be determined mostly by the heat capacity of the small volume in which the power pulse is absorbed as illustrated in Fig. 3(a). In this case a 0.100 milliwatt pulse is 10 ns long which delivers and energy, ΔE, of 1.0 picojoules to the sample. If the size of the volume in which the energy is absorbed is about 0.1 micron or 100 nm and using the heat capacity of silicon 1.63 $J/K\ cm^3$ then the temperature change will be, Fig. 3(a):

$$\Delta T = \frac{\Delta E}{C_p \rho\, s^2 t} = 600^{o} C \tag{11}$$

The small volume will be heated by this short power pulse. The rate of cooling, or the rate at which the volume element is quenched, can be found by an equivalent circuit representation of heat conduction as shown in Fig. 3(b). For oxide, K= 0.014 J/(sec cm K).

Fig. 3(c) illustrates the time dependent temperature of a 100nm cube of silicon in Fig. 3(a) on oxide to a 100 microwatt and 10 nanosecond power pulse. The temperature ramps up by 600C and then decays away in a few microseconds.

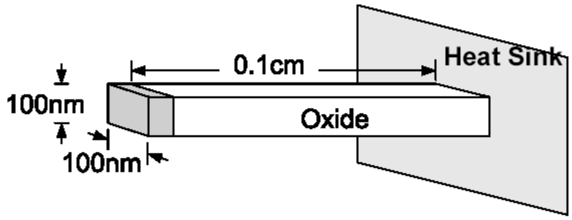

Figure 3(a): Power and heat dissipation in a 100nm cube of silicon on an oxide insulator.

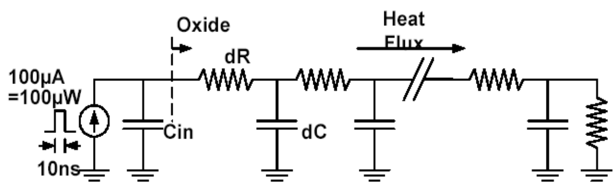

Figure 3(b): Equivalent circuit representation of thermal resistance and heat capacity. A 100 microwatt power pulse is applied for 10 nanoseconds.

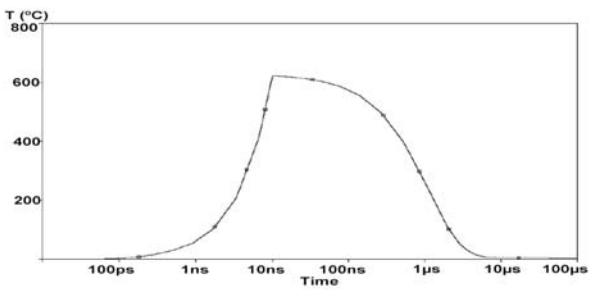

Figure 3(c): Increase of temperature of the silicon device, the temperature ramps up 600C with the power pulse. After the pulse ends the device cools in a few microseconds.

4 CYLINDRICAL COORDINATES

In the cylindrical coordinates, an equivalent circuit representation can be made for heat conduction where for each volume element,

$$R = 1/(K 2\pi l r) \qquad K/Wm \qquad (12)$$

$$C = C_p \rho 2\pi r l \qquad J/K\,m \qquad (13)$$

where K is the thermal conductivity, C_p the heat capacity, ρ the density, and 2πrl the surface area of the cylinder whose radius is r through which there is heat conduction. The steady state or DC resistance of a sample between in an inner cylinder of radius, r1, and an outer radius, r2, is

$$R_{sample} = \ln(r2/r1)/(K 2\pi l) \qquad (14)$$

The time dependent and frequency dependent solutions of the diffusion equation can in cylindrical coordinates can be most conveniently found using equivalent circuit techniques.

Fig. 4 illustrates the application of the technique to a practical device problem in cylindrical coordinates, a nanoscale silicon on insulator transistor Fig. 4(a) driven with an average power of 50 microwatts. The temperature of the device in Fig. 4(c) follows the 100Mhz signal applied in Fig. 4(b) and there is also a steady ramp up in the device temperature. These illustrations show that circuit simulations can quickly and easily provide estimates of the time and frequency dependent temperatures of nanoscale devices. There are no known analytical techniques for the time and frequency dependent solutions of the diffusion equation and in particular for the case of the device in Fig. 4.

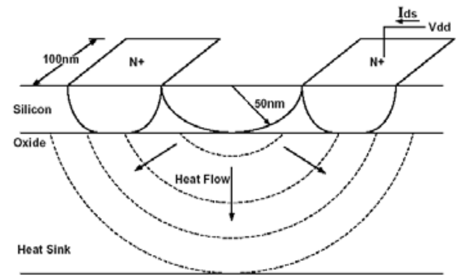

Figure 4(a): Nanoscale silicon on insulator transistor, model in cylindrical coordinates.

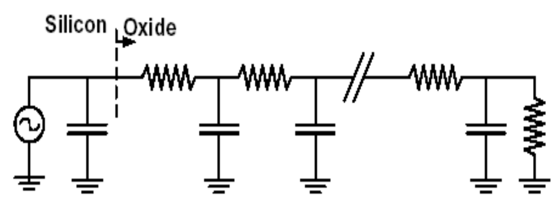

Figure 4(b): Equivalent circuit model with AC power dissipation in the silicon on insulator transistor.

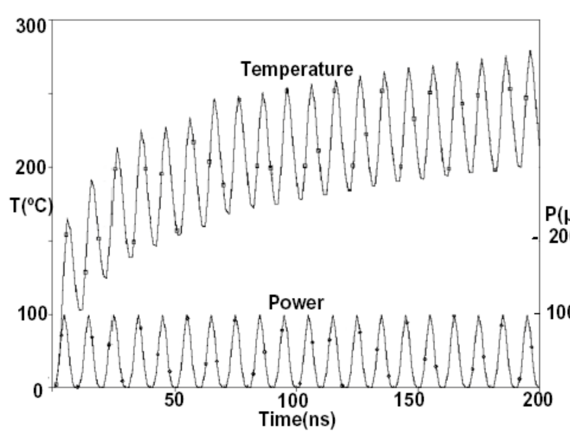

Figure 4 (c): Time dependence of the power dissipation and temperature of the nanoscale device.

5 SPHERICAL COORDINATES

In the spherical coordinates, an equivalent circuit representation can be made for heat conduction in a nanoscale device shown in Fig. 5. The inner silicon sphere in Fig. 5 of radius 100nm represents a device, the outer spherical shell represents heat flow through an oxide insulator to a heat sink. For each volume element,

$$R=1/(K4\pi r^2) \quad \text{K/Wm} \tag{15}$$

$$C=C_p\,\rho 4\pi r^2 \quad \text{J/Km} \tag{16}$$

where K is the thermal conductivity, C_p the heat capacity, ρ the density, and $4\pi r^2$ the surface area of the sphere whose radius is r through which there is heat conduction. Temperature is again analogous to voltage and heat flux analogous to current.

The time invariant steady state, or DC, solution for this line when terminated by a heat sink with infinite heat capacity is then a radial variation in temperature where the heat flux; $flux = K4\pi r^2 \dfrac{dT}{dr}$ and for the total line,

$$flux = \frac{\Delta T}{R_{sample}}$$

where $R_{sample}=\int_{r1}^{r2} Rdr = \int_{r1}^{r2} \frac{1}{4\pi K r^2}dr \approx \frac{1}{4\pi K r_1}$ (17)

The time dependent and frequency dependent solutions of the diffusion equation can again in spherical coordinates be most conveniently found using equivalent circuit techniques. Fig. 6 shows an AC analysis and circuit simulation for the frequency dependence of heat flow in the R-C diffusion line. Frequency components above 10 MHz are strongly attenuated by the heat capacity of the oxide.

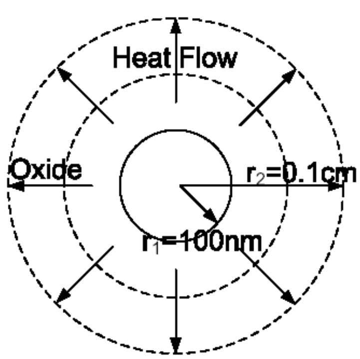

Figure 5: A 100nm nanoscale device surrounded by oxide with radial heat flow.

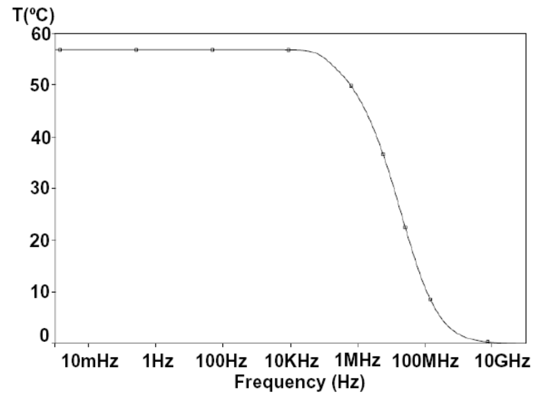

Figure 6: Frequency dependence of temperature variations.

The results of driving the line with an AC power dissipation at 100MHz with an average vale of 100uwatts shows the variations at 100 MHz are attenuated.The temperature of the silicon sample at the center of the sphere can not follow these high frequency variations, however, the average temperature is shown to increase with time. There is a small AC variation in temperature superimposed upon this average value of temperature which increases with time, similar to that in Fig. 4(c).

6 CONCLUSIONS

A textbook formulation of the problem of frequency dependent solutions to the diffusion equation has been given by Kittel and Kroemer [1]. They give no solution to this extremely difficult differential equation and note only that high frequency variations will be strongly attenuated. The equivalent circuit technique allow solutions to be readily obtained by circuit simulations.

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