# Optimization of Piezoelectric Transformers Using Genetic Algorithm

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## Abstract

In this work, we present the optimization work for piezoelectric transformers (PT) by using the elite Genetic Algorithm (GA). A finite-element solver, NTUPZE, is used to calculate the behaviors of piezoelectric transformers. The design parameters are the dimensions of the devices, and the design optimization objects are the device efficiency and gain. Since GA does not require a gradient function for projection and is capable of dealing with multi-objective optimization problems, it is a good optimization approach for this work. The optimal results of Rosen-type and Rosen-modal-type PTs are presented. The optimized efficiencies for both cases are close to 1 with a 100 $k\Omega$ electrical loading connected on the output electrodes.

***Keywords***: Genetic algorithm (GA), piezoelectric transformer, finite element method (FEM), multi-object optimization

## 1 Introduction

The liquid crystal display (LCD) technology has been being very popular for personal/portable computing and is proved to be one of the most promising technologies for the next generation TV. Each LCD panel requires a few cold cathode fluorescent lamps (CCFL) as backlight sources. A CCFL requires a high voltage source for ignition. The conventional way to step up or down AC voltages is to use electromagnetic transformers. For this type of transformers, primary and secondary coils are electro-magnetically coupled through a magnetic core. It is well known that electromagnetic transformers are efficient for power supply applications. However, the energy losses, such as skin-effect loss, thin wire loss, core loss, and hysteretic loss, become significant as the sizes of magnetic transformers are reduced. Therefore, for flat-panel displays, electromagnetic transformers might be too bulky, although they are still used by most LCD panels. Piezoelectric transformers (PT), which were proposed in 1954 by Rosen [1], step up or step down input voltages by electrical/mechanical/electrical conversion. Piezoelectric transformers have been proved to be promising alternatives for magnetic transformers because they have several advantages, such as low profile, high efficiency (low loss), no electromagnetic radiation, nonflammable, low harmonic current noise, low temperature rise, and so on. Furthermore, since the fabrication technology of thin-film piezoelectric materials becomes mature, piezoelectric transformers are particularly useful because they cam be integrated with the MEMS devices that require high input voltages, such as comb drives or electrostatic switches.

In order to design and optimize piezoelectric transformers, a 3-D finite element method (FEM) piezoelectric solver is needed. Typical commercial finite element packages, such as ANSYS and ABAQUS, are capable of to analyzing piezoelectric material, but they cannot take into consideration the electric loading (the loading effect) connected on the output port of piezoelectric transformers. We developed a 3-D FEM piezoelectric solver NTUPZE which employs a new finite-element piezoelectric formulation and is capable of accounting for various types of electric loading conditions. The details of the solver are in [2].

Voltage gain and efficiency of piezoelectric transformers are the major aspects for device operations. Voltage gain and efficiency are influenced by several factors, such as material properties, electrical loadings, electrode designs and geometry sizes. In this work, two optimal design cases are presented. Figure 1 shows the schematic views of a Rosen-type and a Rosen-modal-type [3] piezoelectric transformer that we modeled and measured in this work. The major difference between these two devices is the shapes of the electrodes. The sinusoidal electrode pattern of the Rosen-modal-type device can reject unwanted frequency responses and thus has better performance than the traditional Rosen-type device. In this work, the design goals are the voltage gain and the efficiency. The design parameters are the length, the width and the thickness of the devices. We assume that the input section and the output section have same dimensions. Since the voltage gain and the efficiency are not dependant, this is a multi-object optimization.

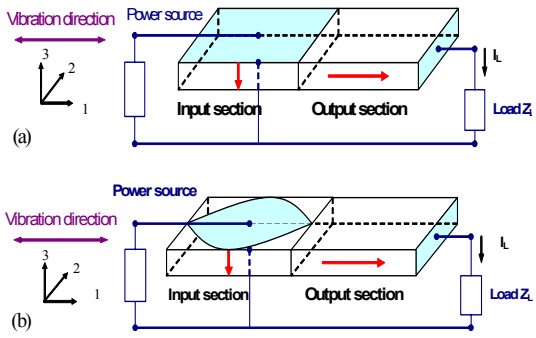


Figure 1 Typical schematic views of (a) a Rosen-type and (b) a Rosen-modal-type piezoelectric transformer with electric loadings imposed at the output electrodes.

Traditional optimization approaches, such as the gradient method, the random search method, and the golden-ratio search method, require analytical models that will be used to evaluate necessary information for search direction [4]. The analytical models of these methods are difficult to obtain in the case when field solvers (FEM or BEM solvers) are employed for performance estimation. Compared with traditional optimization approaches, GA has

the following characteristics. GA works with a coding of the parameter set rather than the parameters themselves. GA searches from a population of points rather than a single point. GA use payoff information (objective function) rather than derivatives or other auxiliary knowledge. GA uses the probabilistic transition rules rather than the deterministic rule [5]. Therefore, GA is suitable for the cases where only little or no prior knowledge (i.e., no analytical prediction of system behaviors) is known. In this work, we primarily use a FEM solver, NTUPZE, to accurately estimate the performance of piezoelectric transformers. Therefore, we choose GA as our optimization scheme for device design.

In addition, standard GA sometimes converges to local optimal points (non-optimal point). The elite GA, one of the variants of typical GAs, can ensure evolutionary processes converge to the global optimum [6]. The basic principle of the elite GA is that the best chromosome of each generation is reserved as one of the chromosomes of next children generation. Our preliminary study shows that when elite GA was not employed, different optima were obtained even in the same search space. Therefore, in this work, we use the elite GA for optimizing piezoelectric transformers.

All of the optimization process is implemented in C++, including the piezoelectric solver NTUPZE and the GA procedure. The brief description of GA will be presented in the next section.

## 2 Genetic Algorithm

The GA procedure consists of the following components: a genetic representation of solutions to the problem, a way to create an initial population of solutions, an evaluation function rating solutions, a reproduction step, and a mutation step. The flowchart of the GA algorithm is shown in Figure 2. The brief description for each step is as follows.

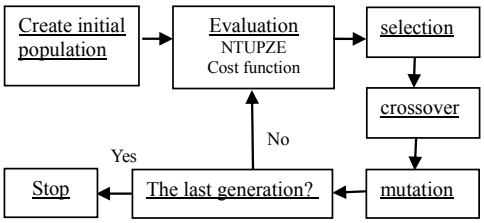


Figure 2 The flowchart of the GA algorithm

*Encoding and Decoding*

The GA works with a coding of the parameter set rather than the parameters themselves. Binary encoding is adopted in this work. There are three parameters in this work. The parameters are in fact the genes; a set of parameters is a chromosome. Figure 3 shows a chromosome composed of three genes (length, width and thickness). Each gene is represented by three bits.

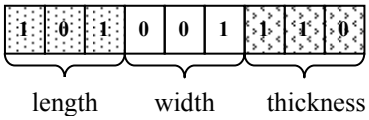


Figure 3 The chromosome composed of three genes

The translation of the encoded chromosome and the design parameters is shown in Equation (1). Note that in Equation (1), *upper* and *lower* are the upper and lower bounds of the design parameter.

$$\text{Design Parameter} = \frac{(upper - lower)}{2} \times \sum_{i=0}^{n} (2^n \times Bit_i) \quad (1)$$

*Creating an initial population*

A population is defined as the collection of all chromosomes in a specific generation in a revolution process. Initial population is created randomly. Determination of the proper size of an initial population for a certain GA process depends on experiments.

*Assigning cost function*

A cost function provides a way to evaluate the performances of chromosomes in a population. The determination of the cost function affects the evolutionary procedure of GA.

In this work, the compromise approach [7] is adopted. The compromise approach calculates the distance between a solution and an ideal solution. The ideal solution is pre-defined by designers. A shorter distance indicates that a better solution is evaluated. The normalized distance between a solution and the ideal solution is defined as (2):

$$r = \frac{\left| z^* - z \right|}{z^*} \quad (2)$$

where $z$ is a computed solution, and $z^*$ is the ideal solution. The cost function is the summation of the product of the each goal's distance and its corresponding weighting, as shown in Equation (3):

$$\text{cost} = w_g \frac{\left| G^* - G \right|}{G^*} + w_e \frac{\left| e^* - e \right|}{e^*} \quad (3)$$

where $G$ and $e$ represent the voltage gain and the efficiency of a piezoelectric transformer, respectively. In this work, the ideal voltage gain is set as 20, and the ideal efficiency is 1.

*Selection*

Selection is the operation that selects chromosomes from the current generation as the parents for next generation. Selection is the driving force in GA. In this work, two different selection methods, Roulette wheel selection and tournament selection [8], are tested.

*Crossover*

Crossover is the GA's primary search routine. The crossover process is to randomly pick a pair of chromosomes as parents. Two children (i.e., two chromosomes) are born by randomly switching portions of the chromosomes between parents.

In the elite GA, the best chromosome of a generation is copied to the next generation. This process guarantees that the best chromosome will never disappear during the evolution process.

*Mutation*

The mutation step is the GA's global search routine. A conventional global optimization typically changes search directions or cost functions to jump out of a local optimum, while the GA employs the mutation operation to avoid the optimal solution trapped in a local optimum. The mutation process is to randomly pick a chromosome and to randomly inverse one of its bits after the crossover. In the binary coding, 1 converts to 0, and 0 converts to 1. The mutation ratio plays an important role in a GA optimization. If mutation ratio is too large, convergence rate will be small. In this work, the mutation ratio is set as 0.08.

## 3 Results and Discussion

In this section, we will present the optimization results for two types of piezoelectric transformers, the Rosen-type and Rosen-modal-type devices. The discussions on the selection methods as well as the influence of weighting coefficients in the cost function will also provided.

### 3.1 Optimization of a Rosen-type PT

In order to obtain a high convergence rate, two selection methods, the Roulette wheel selection tournament selection, are tested. Under the same condition, tournament selection method performs better than the Roulette wheel selection method. After about 30 generations, almost every chromosome converges to the minimum for the tournament selection method, while in Roulette selection method usually takes more than 50 generations for convergence.

Using the tournament selection method, the optimal design of the Rosen-type piezoelectric transformer is obtained with following GA parameters: a mutation rate of 0.08, an encoding of 3 bits per gene. The design space is 30 to 80 mm in length, 4 to 10 mm in width and 1 to 3 mm in thickness. Figure 4(a) shows the deformed shape and the voltage. The convergence curve is showed in Figure 5. Figure 6 shows the curves of the simulated voltage gain and the efficiency vs. frequency. The obtained optimal dimensions of the Rosen-type piezoelectric transformer are 48.75 mm in length, 9.25 mm in width and 2.75 mm in thickness. With an electrical loading impedance of $100\,k\Omega$ the simulated voltage gain and efficiency are 5.26 and 97.53 %, respectively.

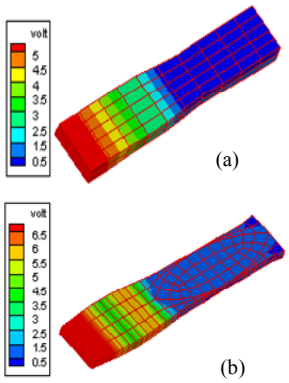


Figure 4 The deformed shape and the voltage of (a) Rosen-type (b) Rosen-modal-type PTs when optimal conditions are obtained.

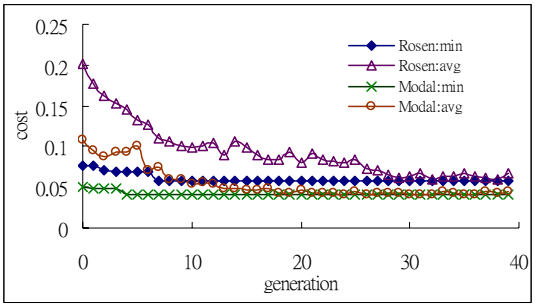


Figure 5 The comparison of convergences between two cases.

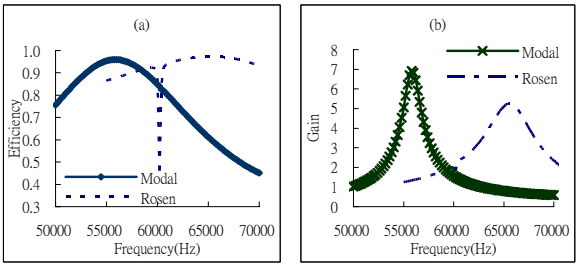


Figure 6 The comparison of simulated (a) voltage gain and (b) efficiency between two cases.

### 3.2 Optimization of a Rosen-modal-type PT

For this case, the first optimal design case (Version 1) is obtained with the following GA parameters: a mutation rate of 0.08, an encoding of 3 bits per gene, and a design space of 60 to 160 mm in length, 4 to 10 mm in width and 1 to 3 mm in thickness. Again, the tournament selection is used. Figure 7 and Figure 8 are the curves of the simulated voltage gain and the efficiency vs. frequency. The obtained optimal dimensions of Rosen-modal-type piezoelectric transformer are 60.0 mm in length, 8.5 mm in width and 2.5 mm in thickness. With an electrical loading impedance of $100\,k\Omega$, the simulated voltage gain and efficiency are 5.5 and 96.1 %, respectively.

We also tried to use different weightings for the cost function in the same design space. Five different sets of weighting coefficients ($w_G$, $w_e$), are used: (0.5, 0.5), (0.7, 0.3), (0.3, 0.7), (0.8, 0.2) and (0.2, 0.8). However, these five sets of weightings give the same optimization results. It implies that the optimized results are much better than the non-optimal results among the chromosomes in the design space. Therefore, the weighting is insensitive for the GA to search for the optimal parameters.

For this case, because the optimized length is right on the border of design space, we redefine the GA parameters for the second design case (Version 2). The modified design parameters are as follows: a mutation rate of 0.08, an encoding of 3 bits per gene, a design space of 24 to 80 mm in length, 4 to 10 mm in width and 1 to 3 mm in thickness. Figure 7 and Figure 8 are the curves of the simulated voltage gain and the efficiency vs. frequency for this case. For a $100\,k\Omega$ loading impedance, the optimal dimensions of the Rosen-modal-type piezoelectric transformer are 55.5 mm in length, 9.625 mm in width and 2.75 mm in thickness. The simulated voltage gain and

efficiency are 4.41 and 96.5%, respectively. Compared with the optimization results of Version 1, the voltage gain is lower, while the efficiency is higher.

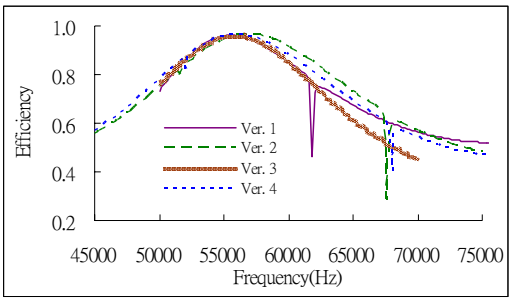


Figure 7 The comparison of efficiencies between optimal designs of a Rosen-modal-type piezoelectric transformer

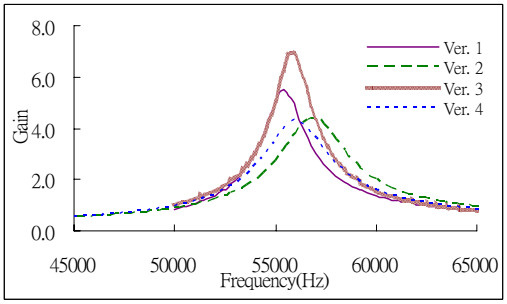


Figure 8 The comparison of voltage gains between optimal designs of a Rosen-modal-type piezoelectric transformer

For the design of Version 3, we increase the resolution of encoded chromosomes. We redefine the GA parameters as follows: a mutation rate of 0.08, an encoding of 4 bits per gene, a design space of 22 to 72 mm in length, 4 to 10 mm in width and 1 to 3 mm in thickness. Figure 7 and Figure 8 are the curves of the simulated voltage gain and the efficiency vs. frequency. For the Rosen-modal-type piezoelectric transformer with an electrical loading impedance of $100\,k\Omega$, the optimal dimensions are 56.375 mm in length, 9.25 mm in width and 2.75 mm in thickness. The simulated voltage gain and efficiency are 6.92 and 95.9 %, respectively. Compared with last two versions, the voltage gain is much higher.

Note that the efficiency of Version 3 is a little lower, while the voltage gain is pretty high. In order to search for a new design with a higher efficiency in this same search space, a set of highly-unbalanced weighting coefficients are used: ( $w_G$ , $w_e$ ) = (0.1, 0.9). The optimal design of Version 4 is obtained with following GA parameters: mutation rate of 0.08, encoding of 4 bits per gene, same design space are that of Version 3. Figure 7 and Figure 8 are the curves of the simulated voltage gain and the efficiency vs. frequency. The obtained optimal dimensions of Rosen-modal-type piezoelectric transformer are 56.375 mm in length, 9.625 mm in width and 2.875 mm in thickness. The simulated voltage gain and efficiency are 4.30 and 96.3 %, respectively. The efficiency is higher than that of Version 3. However, the voltage gain is much smaller.

From the aforementioned discussions, the design of Version 3 is the best design for this case. Figure 4(b) shows the deformed shape and voltage of optimal Rosen-modal-type piezoelectric transformer. Table.1 shows the results and the GA parameters of the optimal designs in both cases.

| Type | | Rosen | Rosen-modal |
|---|---|---|---|
| GA | Chromosome # | 40 | |
| | Generation # | 40 | |
| | Mutation ratio | 0.08 | |
| | Pick ratio | 0.8 | |
| | Gene bit # | 3 | 4 |
| | Cost value | 0.0574 | 0.0408 |
| Optimal design | length | 48.75 mm | 56.375 mm |
| | Width | 9.25 mm | 9.25 mm |
| | thickness | 2.75 mm | 2.75 mm |
| NTUPZE | Load | 100 k ohm | |
| | Max efficiency | 97.33% | 95.9% |
| | Max gain | 5.26 | 6.92 |

Table 1 The results and the settings of the optimal designs in both cases

## 4 Conclusion

In this work, the optimization works for the Rosen-type and the Rosen-modal-type piezoelectric transformers are presented. We use the elite GA with the tournament selection method for the optimization. The finite-element solver, the NTUPZE, is used to calculate the behaviors of piezoelectric transformers. The discussions on the selection rules as well as the influence of weighting coefficients of the cost function were also provided. The optimized efficiencies for both cases are close to 1 with a $100\,k\Omega$ loading.

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