# Theoretical Model for the Nanobundle Network Transistors Below and Above Percolation Limit

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## ABSTRACT

Nanobundle network transistors (NBTs) have emerged as a viable, higher performance alternative to poly-silicon and organic transistors with possible applications in macroelectronic displays, chemical/biological sensors, and photovoltaics. A simple analytical model for I-V characteristics of NBTs (below the percolation limit) and a numerical model (above the percolation limit) is proposed. The physics-based predictive model provides a simple relation between transistor characteristics and design parameters which can be used for optimization of NBTs.

***Keywords***: Carbon Nanotube, Stick Percolation, Network Transistor, Inhomogeneous Percolation Theory, Thin Film Transistor.

## 1 INTRODUCTION

Recently there have been a number of reports regarding a new class of thin film transistors based on percolating network of randomly-oriented, finite-length Silicon nanowires (NW) and Carbon nanotubes (NT) [1-6], as shown in Fig. 1. At densities below the percolation limit ($\rho < \rho_{TH}$ [7]), the transistors may have applications in microelectronics and power transistors with on-current ($I_{ON}$) of the order of milliamps, much higher than a single NW/NT devices but still with on/off ratio ~ 500 [1]. For NT/NW densities above the percolation limit ($\rho > \rho_{TH}$), NBTs have emerged as a viable, higher performance alternative to poly-silicon[8] and organic transistors[9] with possible applications in macroelectronic displays, chemical/biological sensors, photovoltaics[1-6]. Several groups have reported a wide variety of experimental data,

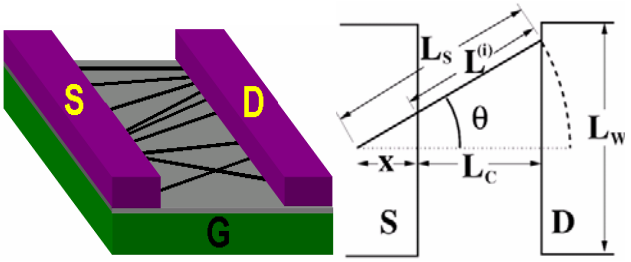


Fig. 1 Geometry of nanobundle network transistor: (a) A schematic of NBT. (b) A nano-stick between the S/D electrodes making an angle $\theta$ with the channel axis.

but there is no unifying theoretical model to explain these recent experiments. In this paper, we propose and validate a simple theory of I-V characteristics of NBTs (below and above the percolation threshold) and use the theory to model recent experiments and assess device performance.

## 2 FOR NBT BELOW PERCOLATION THRESHOLD

### 2.1 Analytical Model

Consider a typical NBT[1, 10] (Fig. 1a) assembled with a bundle of nano-sticks of length $L_S$, isotropically oriented $(0 \le \theta < 2\pi)$ onto gate oxide surface. $L_C$, $L_W$, $L^{(i)}$ and $x$ (Fig. 1b) are channel length, channel width, intercepted channel length for individual sticks and total stick overlap component with the S/D along channel axis, respectively. For $L_C < L_S$ and $\rho < \rho_{TH}$, the tube-tube interaction is unimportant. Assuming uniform probability of germination at all locations, the number of sticks bridging S/D and making an angle between $\theta$ and $\theta + d\theta$ with the channel axis (Fig. 1b and Fig. 2) is given by,

$$dN_S(\theta) = (D_C x)d\theta/(\pi/2) = (2D_C/\pi)(L_S\cos\theta - L_C)d\theta$$

where, $D_C$ is linear tube density and $\cos\theta = (L_C + x)/L_S$. Total number of sticks bridging S/D, $N_S$, is given by summing over all the angles from $0$ to $\theta_{max} = \cos^{-1} L_C / L_S$. Therefore,

$$N_S(R_S) = 2D_C L_S / \pi g_B(R_S)$$

where $R_S = L_C/L_S$, and $g_B(y) = (1 - y^2)^{1/2} - y\cos^{-1}y$.

*Ballistic limit:* For NBTs in short channel limit all the sticks are in ballistic limit and the current is given by,

$$I_B = L_W C_{ox}[V_G - V_{TH}]v_T N_S \qquad (1)$$

where $v_T = (2K_BT/\pi m^*)^{1/2}$ is thermal velocity, $V_{TH}$ is threshold voltage and $C_{OX}$ is gate capacitance. The ballistic current is proportional to number of sticks, as expected.

*Velocity saturation limit:* For NBTs in high bias all the sticks can be in velocity saturation limit so that magnitude of current is independent of individual stick length. Therefore overall transport is again independent of channel length and,

$$I_{sat} = L_W C_{ox}[V_G - V_{TH} - V_D/2]v_{sat} N_S \qquad (2)$$

where $v_{sat}$ is saturation velocity of the carriers.

*Long channel limit:* For NBTs with longer channel limit, the transport in each of the bridging sticks is diffusive and is inversely proportional to intercepted channel length, $L^{(i)}$, for each stick. The current is given by,

$$\frac{I_D}{f(V_D,V_G)} = \int_0^{\theta_S} \frac{2D_C L_C b}{\pi} \frac{(L_S\cos\theta - L_C)}{L_C/\cos\theta} d\theta = \frac{D_C}{\pi} g_D(R_S) \qquad (3)$$

where $g_D(y) = \cos^{-1}y / y - (1 - y^2)^{1/2}$, $\theta_S = \cos^{-1}R_S$ and $f(V_D,V_G) = \mu_\theta L_W C_{OX} [(V_G - V_{TH}) V_D - \beta V_D^2]$[11].

Finally, *for the intermediate channel lengths*, the sticks parallel to channel axis, $\theta \sim 0$, are near ballistic limit and sticks making an angle, $\theta \sim \theta_{max}$, with channel axis are near

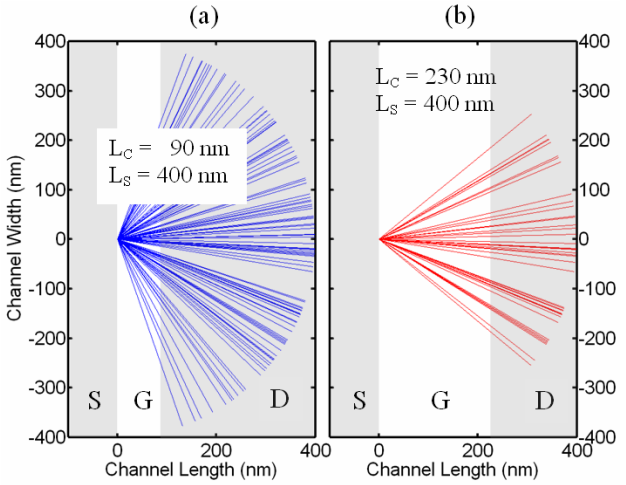


Fig. 2 *Fan Diagram*: If all sticks in Fig. 1a were collected at a point (with angles preserved), it is easy to see that more sticks connect S/D for a transistor with a shorter channel length. Here, stick length is $L_S$ = 400 nm and channel lengths are (a) $L_C$ = 90 nm and (b) $L_C$ = 230 nm.

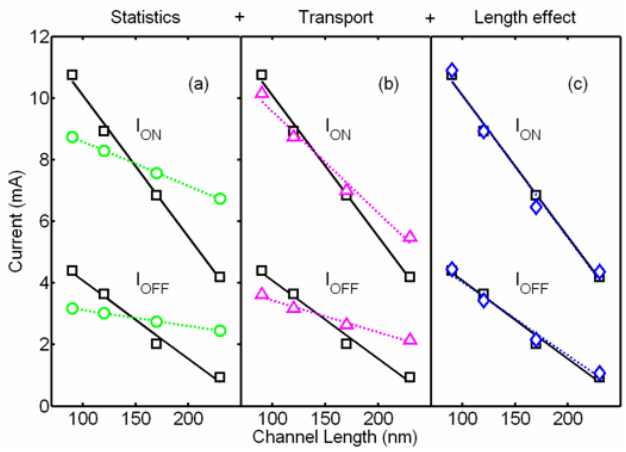


Fig. 3 Experimental[1] (squares with solid line fit) and theoretical (dotted line fits) $I_{ON}$ and $I_{OFF}$ vs. $L_C$. The inverse scaling of $I_{ON}$ and $I_{OFF}$ with $L_C$ is a consequence of sequential addition of (a) *Statistical effect:* Increased probability of sticks to bridge S/D as channel length decreases (Fig 2), (b) *Transport effect:* Prevalence of ballistic transport in shorter channel devices, and (c) *Average length effect:* Average length for m-SWCNT $\langle L_M \rangle \sim$ 350 nm is made less than s-SWCNTs $\langle L_S \rangle \sim$ 1000 nm. Here, mean free path for s-SWCNT $\lambda_D \sim$ 200 nm. Note that theory matches well with experiment only after all three effects are included in the simulation as shown in (c).

diffusive limit. And the proportion of the two types of sticks changes with applied bias. Still, the I-V characteristics of any single nanostick can be described by a simple analytical expression based on a transmission point of view[12]. An exact analytical result for NBTs can be obtained by replacing $1/L^{(i)}$ with $1/(L^{(i)} + \lambda)$where $\lambda$ = min ($\lambda_D$, $\lambda_{sat} = V_D \mu_\theta / v_{sat}$) and $\lambda_D$ is the mean free path for carriers. Hence, the total current

$$\frac{I_T}{f(V_G,V_D)} = \int_0^{\theta_S} \left( \frac{2D_C}{\pi} \frac{L_S\cos\theta - L_C}{L_C/\cos\theta + L_C b} \right) d\theta \qquad (4)$$
$$= \frac{2D_C}{\pi b^2} \left[ b g_B(R_S) - \cos^{-1}R_S + \frac{2(bR_S+1)}{\sqrt{b^2-1}} \tanh^{-1} \frac{(b-1)\tan(\theta_S/2)}{\sqrt{b^2-1}} \right]$$

where, $b = \lambda / L_C$ (Fig. 2a). As $\lambda \to 0$, $I_T \to I_D$ and as $\lambda \to \infty$, $I_T \to I_{sat}$.

## 2.2 Discussion

The model helps explain the inverse scaling of $I_{ON}$ vs. $L_C$ and $I_{OFF}$ vs. $L_C$ (Fig. 3) in the recent measurements of CNT-NBTs [1] as a combination of three factors associated with random distribution of tubes: (1) *Statistical effect:* Increased probability of sticks to bridge S/D as channel length decreases (Fig. 2, 3a), (2) *Transport effect:* Prevalence of ballistic transport in shorter channel devices (Fig. 3b), and (3) *Average length effect:* Average length for metallic (m)-SWCNT $\langle L_M \rangle \sim$ 350 nm being less than semiconducting (s)-SWCNTs $\langle L_S \rangle \sim$ 1000 nm (Fig. 3c). Fig. 5d and 5e show the relative contribution from the three effects. The total statistical effect, which is sum of (1) and (3), plays a dominant role in interpreting this experiment. Here, mean free path for s-SWCNT $\lambda_D \sim$ 200 nm.

# 3 FOR NBT ABOVE PERCOLATION THERSHOLD

## 3.1 Numerical Model

For NBTs with $L_C > L_S$ and density higher than percolation threshold (Fig 4a) the stick-stick interaction is important and analytical solution is not possible. Therefore, $n(V_D, V_G)$ must be determined self-consistently by solving the drift-diffusion equations (appropriate for $L_C > 1\ \mu m$) and the Poisson equation which are generalized for NBT as:

$$\left.\begin{array}{l} \frac{d^2\Phi}{ds^2} + \frac{\rho}{\varepsilon} = 0 \\ \nabla.J_p = 0 \\ \nabla.J_n = 0 \end{array}\right\} \rightarrow \begin{array}{l} \sum_{i=1}^{N} \left( \frac{d^2\Phi_i}{ds^2} + \frac{\rho_i}{\varepsilon} - \frac{(\Phi_i - V_G)}{\lambda^2} + \sum_{j\neq i} \frac{(\Phi_j - \Phi_i)}{\lambda_{ij}^2} \right) = 0, \\ \sum_i \left( \nabla.J_{pi} + \sum_{j\neq i} C_{ij}^p (p_j - p_i) \right) = 0, \\ \sum_i \left( \nabla.J_{ni} + \sum_{j\neq i} C_{ij}^n (n_j - n_i) \right) = 0, \end{array} \qquad (5)$$

where, $N$ is total number of NT/NW, $s$ is in the direction of individual NW/NT, $\rho(x, y)$ is total charge density, and the term $-(\Phi - V_G)/\lambda^2$ (the well known parabolic approximation [13, 14]) introduces the effect of back gate, where $\lambda$ is effective screening length. The term $(\Phi_j - \Phi_i) / \lambda_{ij}^2$ is stick-stick interaction with screening length $\lambda_{ij}$ where a node on

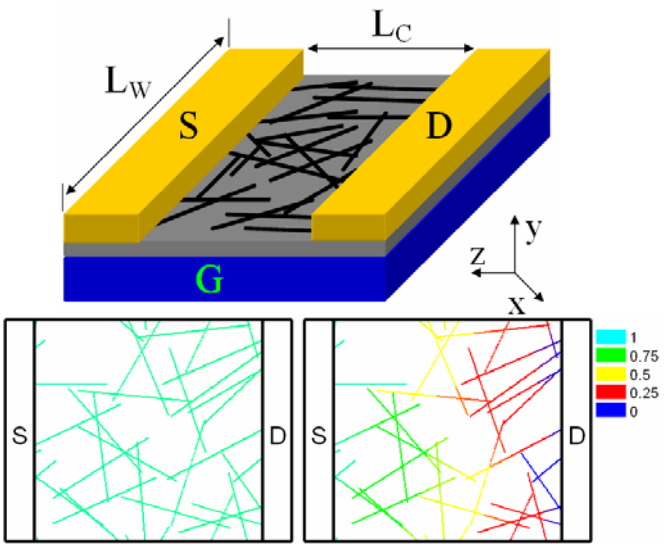


Fig. 4. (a) A schematic of NBT: Geometry of nanobundle network transistor. Distribution of normalized conductance along the sticks in the random network in (b) low and (c) high bias, respectively. The conductance is uniform along the channel in low bias while it varies nonlinearly in the high bias condition resulting in an inhomogeneous percolating network and the contours of conductance are not straight lines parallel to S/D as in conventional MOSFET.

stick $i$ intersects a node on stick $j$. The intersecting nodes act as tiny gates for each other modifying the potential and carrier concentrations [15]. Further, transport is essentially 1D (along the tube) with the additional term $C_{ij}^{n}\ (n_j - n_i)$ in the continuity equation representing charge transfer between nanosticks at the point of intersection. Here higher value of $C_{ij}^{n,p} = G_0/G_1$ implies better electrical contact, where $G_0$ and $G_1$ is mutual and self conductances of the tubes [16].

Doped S/D contacts [17] are assumed to simulate NBT using the Eq. (5) and the initial guess is the equilibrium solution. An iterative self consistent solution is found using Scharfetter-Gummel method of discretization [18]. In each of the iterations, an array, $k$, is filled with 3 elements (corresponding to 3 unknowns $\Phi$, $n$, and $p$) for each node on all the tube with the current solution substituted into LHS of the Eq. (5). If the tube is touching S/D contacts a constant potential boundary condition is imposed otherwise a floating boundary condition is imposed. Then a 2-D sparse derivative matrix, $J$, is populated with mostly lower sub-diagonal entries except a few off-diagonal terms corresponding to the tube-tube interaction. Finally, at the end of each iteration, the correction to potential and charge distribution is obtained as, $dk = J^{-1} k$ and the simulation is stopped when correction to each node is below a threshold value.

## 3.2 Discussion

It was shown in section 2 above that in the 'short-channel limit of $L_C < L_S$ and at low stick density, the NW/NTs behave as individual transistors connected in parallel and therefore the ratio of the $I_D$ for any two bias points is independent of the geometry of the NB-TFTs. For $L_C > L_S$ the numerical simulations [19] show that the scaling relationship is still valid and current can be written as

$$I_D = \frac{A}{L_S}\xi\left(\frac{L_S}{L_C}, \rho_s L_S^2\right) \times f\left(V_G, V_D\right), \qquad (6)$$

where the proportionally constant $A$ depends on oxide capacitance $C_{ox}$, tube diameter $d$ [20], and stick-stick interaction parameter, $C_{ij}^{n,p}$. And $\xi$ and $f$ are functions of geometrical parameters ($L_S$, $L_C$, $\rho_S$) and bias conditions ($V_D$, $V_G$), respectively. We find that the geometrical factor $\xi$ can be written as

$$\xi\left(\frac{L_S}{L_C}, \rho_s L_S^2\right) = \left(\frac{L_S}{L_C}\right)^{m(\rho_s L_S^2)} \qquad (7)$$

where $m$ is a universal exponent of stick percolating system. Fig. 5(a) shows the $I_D$-$V_D$ characteristics of two different random networks for $V_G - V_{TH} = 1.0$ $V$, $L_S = 1$ $\mu m$ and $L_W = 200$ $\mu m$. Fig 5(b) compares the current vs. normalized channel length characteristics from the analytical ($L_C/L_S < 1$) and numerical ($L_C/L_S > 1$) models. The analytical model ignores tube-tube interaction and is only valid for $L_C/L_S << 1$, while the numerical model assumes that the current through each tube is limited by diffusive transport and hence is only valid for $L_C >> \lambda$, the mean free path for the carriers.

For densities much higher than percolation threshold ($\rho_S L_S^2 >> \rho_{th} = 4.236^2/\pi$ [7]), the network behaves as a 2D conductor with $m = 1$. Together with Eq. (7) below we find that Eq. (6) reduces to the classic 'square law' as expected. But for densities near percolation threshold ($\rho_S L_S^2 \sim 4.236^2/\pi$) the exponent takes the value $m \sim 1.8$ (Fig. 5b). Moreover, Fig. 5a shows that the bias-dependent scaling

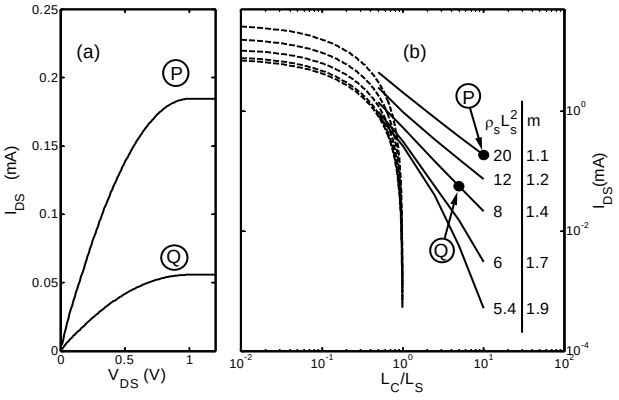


Fig. 5: (a) The source-drain current vs. $V_{DS}$ for $V_G - V_{TH} = 1.0$ $V$ for two networks $P$ and $Q$ with different densities and $L_C$ (as shown in b) here, $L_S = 1$ $\mu m$ and $L_W = 200$ $\mu m$. (b) The on-current vs. normalized channel length computed by analytical model for $L_C < L_S$ (Eq. 4, dotted curves) and numerical model (Eq. 6, solid lines) for $V_G - V_{TH} = 1.0$ $V$ and $V_D = 1$ $V$.

function

$$f\left(V_G, V_D\right) = \left[ (V_G - V_{TH}) V_D - \beta V_D^2 \right] \quad (8)$$

is independent of geometrical parameters ($\beta$~0.5), again satisfying Eq. (6). At very low densities ($\rho << \rho_{th}$), where system behaves as an independent collection of 1D (long-channel) conductors and at very high densities ($\rho >> \rho_{th}$), where the percolating network approximates a classical 2D homogenous thin film limit, the voltage scaling function $f(V_G,V_D)$ would follow the classical 'square-law' formula. But, for $\rho \sim \rho_{th}$, the constant conductivity contours in high bias regime may not be the lines perpendicular to channel axis (Fig. 4b), linear percolation theory may not be applicable and a classical MOSFET like behavior may not be expected. But surprisingly, our simulations show that Eq. (6) holds for arbitrary stick density above and below the percolation threshold. This robust density independent voltage scaling may be arising from the fact that, given the 1D approximation to Poisson equation in (1), any random network of sticks can be reduced to a single (homogeneous) thin-film strip of effective width $W$.

The implication of our results is that once $A$, $V_{TH}$ and $\beta$ is determined (for Eq. 6) by $I_D$-$V_D$ and $I_D$-$V_G$ measurement and $m$ is determined from Fig. (5b) for particular $L_S$, $L_C$, and $\rho_S$ for one of the transistors, one can readily determine the transistor performance of any other transistor of arbitrary $L_S$, $L_C$, and $\rho_S$, (Fig. 5b) and biasing condition reducing the technology development and characterization time significantly. Second, Eq. (6) provides a *'bottom-up' definition of effective-mobility*, $\mu_{eff} \sim (dI_D/dV_G/V_D)(L_S/\xi)/(L_W C_{OX})$, the value of which is independent of $L_C$ and can be used to compare experimental data from various laboratories. For very high density networks, $\mu_{eff}$ reduces to conventional mobility equation as $m = 1$ and $\xi = L_S / L_C$ in Eq. (7).

## 4 CONCLUSIONS

We have proposed a simple analytical model for I-V characteristics of NBTs (below the percolation limit) and a numerical model (above the percolation limit). We have generalized the linear stick percolation theory to nonlinear regime to find a scaling formula (6) to compute $I_D$-$V_D$ characteristics of NB-TFT that, once calibrated, can be used to establish performance limits of NB-TFTs of arbitrary geometry and operating conditions. Our analysis therefore would help organize experimental data from various research groups and could have significant impact on the development of NB-TFT technology.

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