Combination of Analytical Models and Order Reduction Methods for System Level Modeling of Gyroscopes

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ABSTRACT

Within the design process of complex sensor systems different physical effects and electronic circuits for signal processing have to be analyzed. A simulation of the entire system including models for micromechanics and field effects based on partial differential equations is often impossible or too expensive concerning computing time. Furthermore, many details of component behavior calculated by Finite Element Method (FEM) are not needed at system level. In this paper we describe an approach for system level simulation of sensor systems and its application to a gyro sensor system.

Keywords: gyroscope, system level simulation, model order reduction, model generation, HDL

1 INTRODUCTION

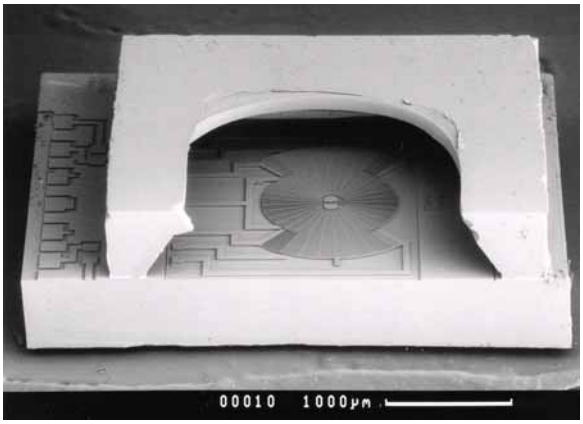
In the design process of microsystems the integration of micromechanics and microelectronics to a dedicated function is a crucial task. Especially the design of signal processing algorithms and electronics must be supported by system level simulations of the entire microsystem. Due to the heterogeneous structure of microsystems it is necessary to deal with different types of models:

- conservative systems, i.e. with energy transport between subsystems, such as electrical or mechanical networks,
- non-conservative systems like control block diagrams, and
- time-discrete systems like digital circuits.

Description of model behavior can be based on equations derived from physics [1, 2, 3] or on approximation of measured data. Futhermore, model order reduction methods may be applied to derive models from FEM equations [4, 5, 6]. The implementation of these different model types is well-supported by languages like VHDL-AMS or Verilog-AMS. But, for system design also MATLAB/SIMULINK is widely used. In this paper we describe the application of a modeling approach for microsystems [7] to a gyro sensor system. This approach is based on a unified model interface and a unified structure of model equations. It allows the above mentioned combination of model types and is suitable for different tool environments.

2 GYRO SENSOR SYSTEM

The gyro sensor system (gyroscope) consists of a polysilicon sensor element (Figure 1) and an electronic circuit for excitation of the structure and for evaluation of the sensor signal (Figure 2). The sensor element is fixed in the center by flexible beams and is excited electrostatically via



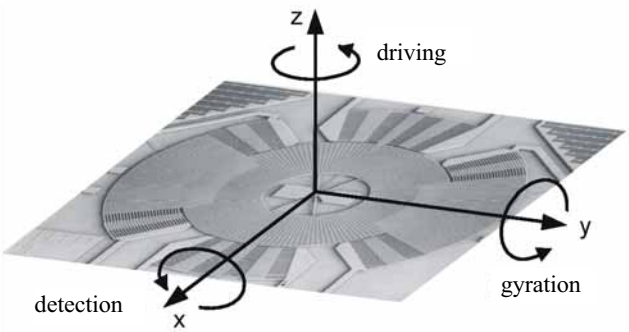


Figure 1: Left: packaged gyro sensor, right: detailed view of one comb drive structure

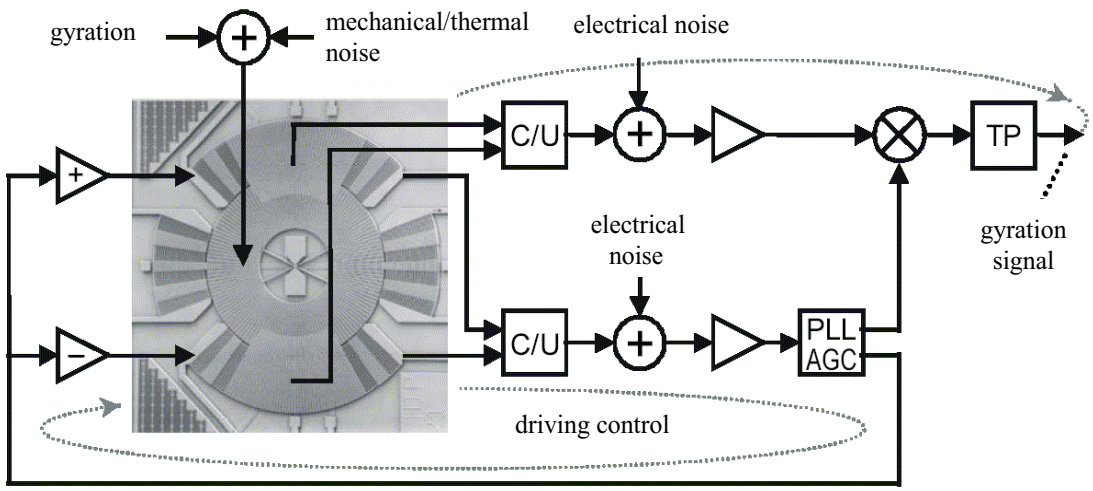


Figure 2: Sensor system including mechanical part, driving control, and signal processing

comb drives to perform resonant oscillations in the xy plane. Due to the law of conservation of angular momentum a gyration about the y-axis leads to an orthogonal oscillation about the x-axis. The resulting change in the distance between sensor element and substrate is evaluated as a capacitive sensor signal.

3 MODELING OF MICROMECHANICAL INERTIAL SENSORS

For system level simulation of the sensor system we have to consider the behavior of the large seismic mass, its coupling to the electrical domain via comb drive structures, the influence of the suspension and the feedback via the detection electrodes. All these effects have to be analyzed in their interaction and they have to be coupled with electronics. Furthermore, we have to deal with linear or non-linear characteristics of the mentioned physical effects. Thus, we divided the gyro sensor model into different parts which were modeled separately and, finally, connected to the model of the entire system.

Deflections and rotations at selected points of the sensor as well as the most important eigenfrequencies should be reproduced to support the design of algorithms for driving and detection (see Figure 2).

3.1 Seismic Mass

The mechanical part of the gyro sensor, disregarding the suspension beams, was modeled using the FEM simulator ANSYS. A set of about 110,000 equations was exported from the simulator. These equations have the form:

$$M\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + D\frac{\mathrm{d}x}{\mathrm{d}t} + Kx = Bf \qquad x_a = B_a^{\mathsf{T}} x \tag{1}$$

The matrices M, D, and K are the mass, damping, and stiffness matrices with a high dimension N. B is the incidence matrix of points with applied forces. Mostly, in system simulation only the behavior of a component at certain points x_a is relevant for the overall system performance. For components in system simulation the terminals are also the observation points, $B = B_a$.

Using a projection method [5] we transform the above system into a low-dimensional subspace

$$V^{\mathsf{T}}MV\frac{\mathrm{d}^2\tilde{x}}{\mathrm{d}t^2} + V^{\mathsf{T}}DV\frac{\mathrm{d}\tilde{x}}{\mathrm{d}t} + V^{\mathsf{T}}KV\tilde{x} = V^{\mathsf{T}}Bf \qquad x_a = B_a^{\mathsf{T}}V\tilde{x} \tag{2}$$

The projection matrix V can be constructed using several techniques. E.g., it can consist of the „most important“ eigenvectors of (1). The dimension of the reduced system is $n<<N$. From the solution $\tilde{x}$ of the reduced system the solution $x = V\tilde{x}$ of the original system can be derived. For first order systems Krylov subspace methods have become popular during the past years. Recently, the ENOR algorithm has been developed for second order systems (1). A modification of this algorithm was investigated by the authors [5].

The demands on the behavioral model of the seismic mass are to reproduce the deflections and rotations at selected points as well as the most important eigenfrequencies. Using the ENOR algorithm we obtained a reduced, linear system of dimension n=20 which was used to generate the corresponding behavioral model.

3.2 Suspension

Four suspension beams connect the seismic mass with the substrate [8] (Figure 3). They have a quite simple geometry but they show non-linear behavior due to large deflections.

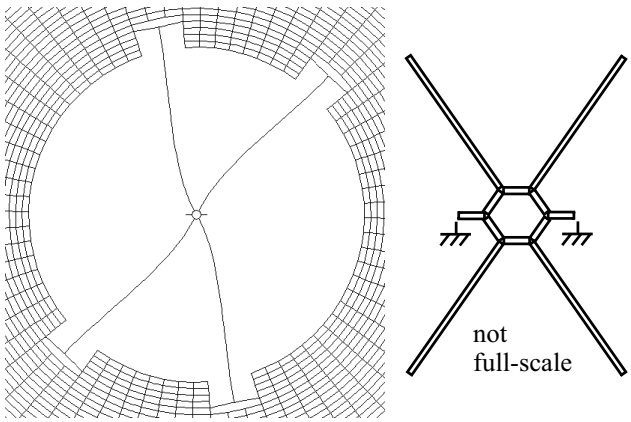


Figure 3: Suspension beams, left: FE model showing large deflections, right: connection of beam models

Analytical beam models with six terminals per beam end are used. These six terminals are corresponding to the three translational and the three rotational degrees of freedom in 3D. The across quantities are the displacements along the main axes and the rotations about these axes, the through quantities are the forces and torques, respectively.

The terminal behavior of a beam element is described by the following equation of movement:

$$M \cdot \frac{d^2}{dt^2}x + D \cdot \frac{d}{dt}x + K \cdot x = f \tag{3}$$

f is the vector of the through quantities and x represents the vector of the across quantities at the terminals. M, D and K are the mass, damping and stiffness matrices of the beam element. Taking the position of the beam ends, the geometric and material properties into account, the system matrices are determined internally and the equations of motion are solved. Geometric non-linearities are considered by evaluating changes of the beam length.

The behavioral model of the seismic mass uses the same six degrees of freedom like the analytical models. This enables us to connect both kinds of models easily.

3.3 Electrostatic Effects

To compute the capacity and electrostatic forces between the detection electrodes and substrate behavioral models considering electrostatic effects are needed. Due to the regular and recurring perforation of the rotor structure it makes sense to break it down to generic structures (unit cells, see Figure 4) [8, 9]. These models are implemented in special electromechanical transducer elements and represent the behavior of a small area of the perforated rotor structure using the methods of conformal mapping. The entire electrodes are formed by parallel connection of a multitude of such unit cells. To consider higher modes of the seismic mass the deflections of certain points on the structure are evaluated and used to calculate the curvature of the electrodes and, thus, to parametrize the unit cells.

Parameters of a cell are the geometric and material properties, input quantities are the electrical voltage and the displacement across the electrodes, and output quantities are the capacity and the electrostatic forces between the electrodes. These electrostatic forces are added up and fed in to the additional points on the structure. The total capacity is calculated and used as input to the signal processing blocks of the sensor system.

4 SYSTEM SIMULATION OF GYRO SENSOR

The impact of the non-linear behavior of the suspension beams is shown in the force-displacement-characteristic in Figure 5, left. The force was applied in the driving direction of the sensor at the comb drives and the displacement was determined in the same point. The simulation results of the small signal response for lower eigenfrequencies show a very good correspondence between the full FE model and the reduced system (Figure 5, table). For transient simulation (Figure 5, right) the sensor was stimulated with a trapezoidal gyration shortly after reaching the steady state in its driving mode. The oscillating movement can be observed as a change in capacitance at the detection electrodes.

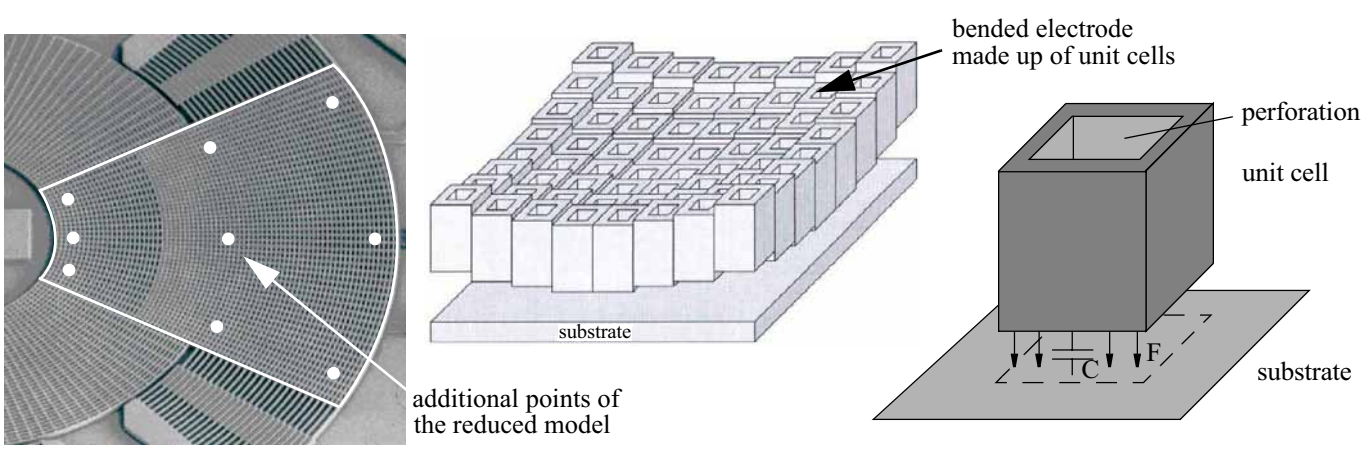


Figure 4: Left: one detection electrode of the inertial sensor, middle: possible deformations of the electrode, right: unit cell

5 CONCLUSIONS

The introduced method allows to generate models for system simulation of complex sensor systems including electrostatic effects and non-linear behavior of the suspension beams.

These models can be combined with models of electronic circuits for signal processing. Thus, a system level simulation of the entire sensor system is possible. Furthermore, the modeling approach allows the integration of additional effects, like damping, by connecting their behavioral models to the mechanical structure. Combining all these parts to a design methodology, the design of the signal processing can be supported effectively.

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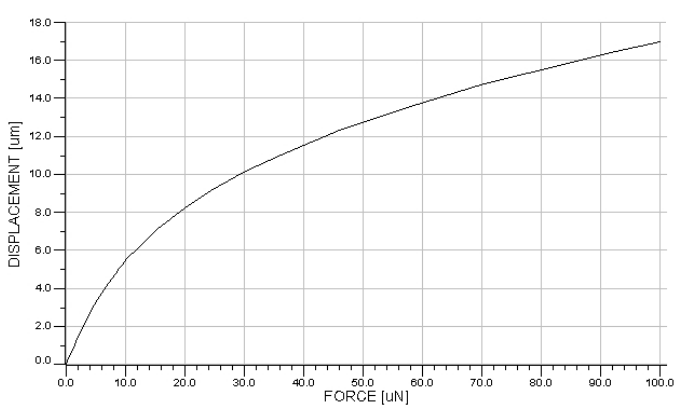

eigenfrequencies	F_1 [Hz]	F_2 [Hz]	F_3 [Hz]
ANSYS	1442	2422	4378
reduced system	1442	2421	4375

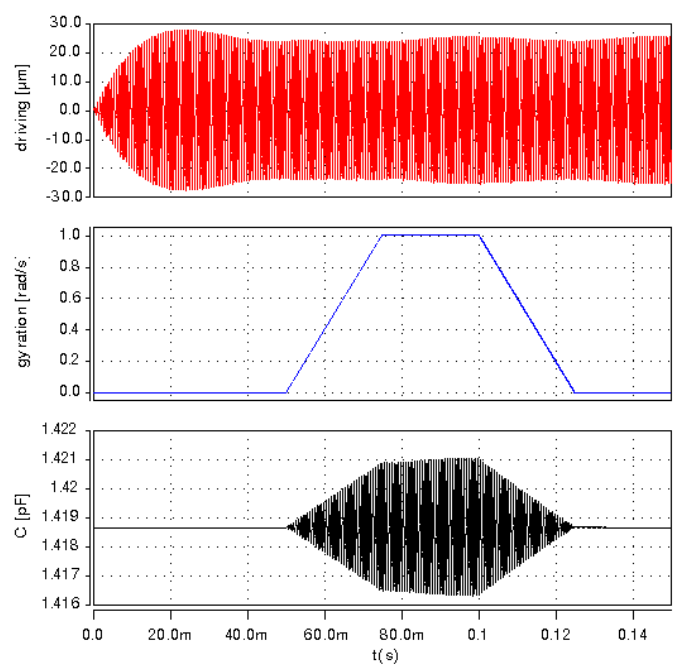

Figure 5: Left: non-linear force-displacement-characteristic, eigenfrequencies – reduced system compared to the full system right: transient analysis – driving of the sensor, acting gyration and resulting capacity