Effects of etch holes in microelectromechanical resonators

Jize Yan and Ashwin A. Seshia
Department of Engineering, University of Cambridge, UK

ABSTRACT

Micromachining processes such as surface machining of polysilicon microstructures or SOI based processes often require etch holes to be created in the structural layer for purposes of releasing the mechanical structures from the substrate. The present work examines the effect of the nature and density of etch holes on the resonant frequency of micromechanical resonators. Effects relating to geometry, dimension and arrangement of etch holes are considered. Two predictive analytical models for estimating resonant frequency are considered and the results compared with finite element simulations on beam structures incorporating etch holes. The results from using these models can be used to formulate design rules for the size and arrangement of etch holes in micromechanical resonators.

Keywords: resonators, etch holes, frequency references.

1 INTRODUCTION

Silicon micromechanical resonators have been demonstrated as building blocks for variety of signal processing [1] and sensor applications [2]. For instance, silicon micromechanical resonators can be used as frequency references in oscillators [3] where the high-quality factor resonant characteristics of the micromechanical element are utilized to preferentially shape the output energy at a fixed frequency, usually either equal to the resonant frequency of the micromechanical element or a suitable higher harmonic. In a resonant sensor application [2], the shift in the resonant frequency of the micromechanical element is detected to give a precise estimate of the measurand. Thus in all of these applications a good starting estimate of the resonant frequency factors strongly into design considerations.

Silicon micromechanical resonators are implemented commonly in surface micromachining processes or other processes where etch holes are required to be created in free-standing structures. These etch holes are required for purposes of releasing the structure for free motion following process completion. Process-induced offset in resonant frequency of micromechanical structures due to change in the overall dimensions of the beams (e.g. due to over- or under-etch) is usually amenable to relatively straightforward analysis. However, the effects of etch holes built into these structures are often significant and can play an important role in determining resonant frequency.

Previous work in examining effects of etch holes on MEMS devices include the effect of etch holes on mechanical properties of thin films [4], the electromechanical behaviour of MEMS actuators [5] and the modeling of optical [6] and ferromagnetic MEMS [7]. We build upon the previous work to construct two analytical models to predict the process-induced offset in resonant frequency in free standing structures with etch holes. The first model is based on the Rayleigh-Ritz method [8] for resonant frequency estimation while the second method is based on the effective density model [9]. The analytical results are presented for a variety of resonator topologies and compared to results from finite element simulations.

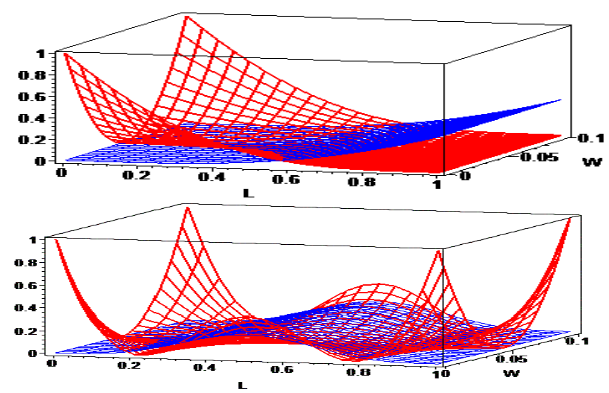


Figure 1: Normalised kinetic energy (blue) and potential energy (red) curves computed analytically for clamped-free beams (a) and clamped-clamped beams (b) plotted as a function of the in-plane dimensions.

2 SMALL HOLE MODEL

A general analytical formulation based on the Rayleigh-Ritz principle is used as a starting point for the 'Small Hole Model'. Resonant frequencies for microstructures of a given topology can be computed if the corresponding mode shape is known and an analytical expression exists for the mode shape. We can solve the differential equations for beam bending with the right boundary conditions in place to obtain a solution for the desired mode shape. We express the kinetic energy and potential energy as a function of beam geometry, material properties and normalized mode function. Figure 1 is a plot of the normalized kinetic energy and potential energy corresponding to the first mode of clamped-free and clamped-clamped beams plotted as a function of the length and width dimensions.

The Rayleigh-Ritz principle for resonant frequency estimation states that the maximum kinetic and potential energies are equal for a vibratory system describing simple harmonic motion. Hence an estimation of the natural frequency is obtained as:

$$M_{eff} = \begin{cases} \int \rho w(x)^2 dV = \int\limits_{total} A w^2(x) dx - \int\limits_{holes} A w^2(x) dx \\ \int \rho u(x)^2 dV = \int\limits_{total} A u^2(x) dx - \int\limits_{holes} A u^2(x) dx \end{cases} \quad (1)$$

$$K_{eff} = \begin{cases} \int EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx = \int\limits_{total} EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx - \int\limits_{holes} EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \\ \int E\left(\frac{\partial u}{\partial x}\right)^2 dV = \int\limits_{total} EA\left(\frac{\partial u}{\partial x}\right)^2 dx - \int\limits_{holes} EA\left(\frac{\partial u}{\partial x}\right)^2 dx \end{cases} \quad (2)$$

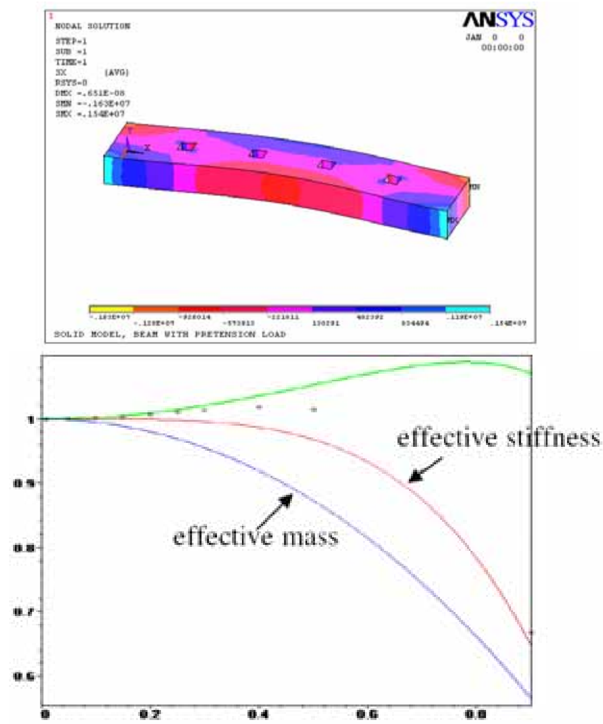
$$\omega = \sqrt{\frac{K_{eff}}{M_{eff}}} \quad (3)$$

Here, *w(x)* represents the normalized analytical solution for transverse or flexural mode resonance mode shape while *u(x)* represents the normalized analytical solution for bulk mode resonance for a beam structures with defined boundary conditions (fixed-fixed, free-free, fixed-free topologies). An effective stiffness and an effective mass for a resonator with etch holes is computed as shown in equations (1) and (2) respectively. As part of the analytical formulation, it assumed that the mode shape solutions for these resonators remain unchanged due to the presence of etch holes. This is a reasonable assumption if the density and number of etch holes is small.

Results from the analytical model are compared with finite element simulations for two common topologies of micromechanical resonators: namely the clamped-clamped beam and the clamped-free or cantilever beam topologies. The first study examined the impact of varying the etch holes size relative to beam width. We notice that effective stiffness and effective density both generally decrease as a function of etch hole density. However, it is the relative magnitude of the shift that defines the trend in resonant frequency.

We examined the impact of varying the etch holes size relative to beam width. The analytical approach is compared to finite element simulation results for a multitude of resonator topologies for varying etch hole size and location. Figure 2 shows a sample comparison of the results for a particular cantilever structure where etch hole size is varied as a fraction of the beam width. It can be seen that the results agree to better than 5% for etch hole sizes of roughly half the beam width. In general, the variation in effective stiffness is more pronounced as the etch hole size increases and the downward shift in resonant frequency more pronounced.

The location of etch holes along the length of the beam also has a pronounced effect on resonant frequency. Figure 3 plots normalised resonant frequency as a function of etch hole location for the first two bending modes of a clamped-clamped beam and a clamped-free beam. Etch holes located at the nodal points do not significantly affect the resonant frequency while holes located at the antinodes show a pronounced effect as would be expected. The shift in resonant frequency is greater for larger etch hole sizes and it can be shown that for the flexural mode resonators that the frequency shift is dependent on the fractional area of the holes.



Hole size normalized to beam width

Figure 2: Comparison of analytical model of normalized resonant frequency shift (y-axis) with FEA simulations for holes of varying sizes. The top curve is the result obtained analytically while the dots indicate the FEA result

3 EFFECTIVE MEDIA MODEL

The 'small hole' model is a useful method to estimate resonant frequencies in the limit that the etch hole density is small and serve as the basis for qualitative design decisions on etch hole location and distribution. However, the model is not particularly suited to hand analysis and the predictive power reduces as the number of etch hole density increases. We present an alternative model for resonant frequency estimation termed the 'Effective Media Model' that serves as the basis to make design decisions on the shape and arrangement of etch holes for known process conditions. The effect of flaws and dislocations introduced in the material by the etch holes is not included in this analysis. However, this effect could potentially be parametrised and lumped into the describing terms for this model.

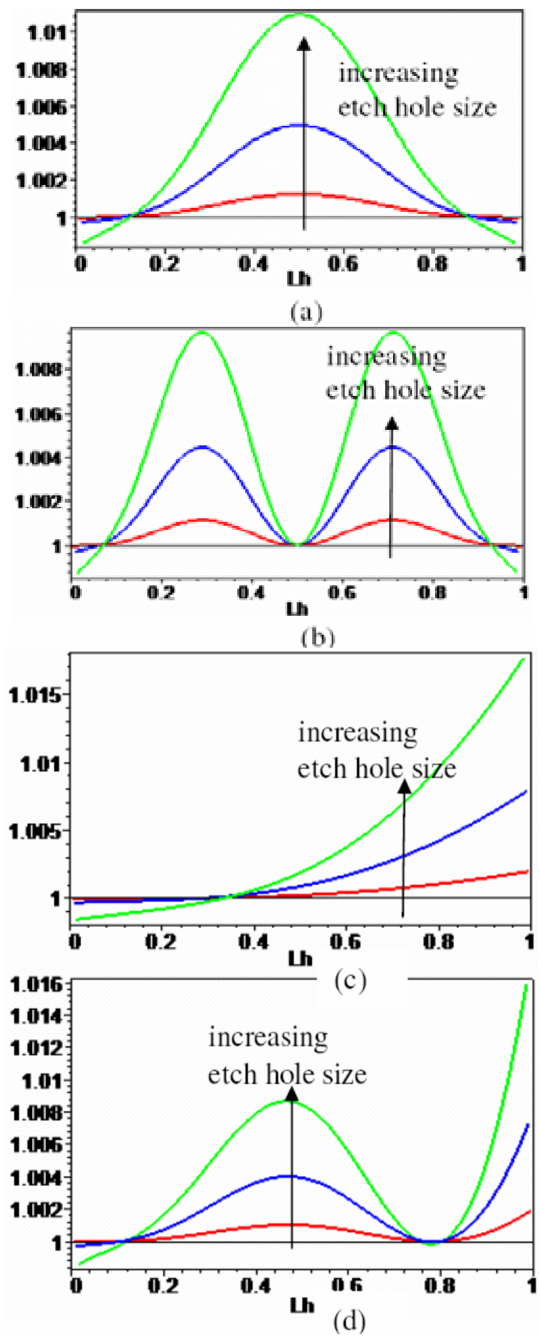


Figure 3: Normalised resonant frequency shift versus location of the hole (Lh) along the length of the beam for clamped-clamped beams (first mode – (a), second mode – (b)) and clamped-free beams (first mode – (c)) and second mode (d)). Curves for varying etch hole size are plotted.

In the effective media model, the resonant frequency is written as:

$$\omega = c\sqrt{\frac{E^*}{\rho^*}} \tag{4}$$

where c is a constant that can be computed for a given mode shape that is a function of geometry, mode shape and boundary conditions for the beam structure. The effect of the etch holes is captured in a new effective Young's Modulus (E^*) and an effective density (ρ^*). The Young's Modulus and the effective density are estimated based on the relative density of etch holes in the beam. For this purpose, we define a process parameter, η, as the ratio of the etch hole area (A_h) to the squared distance between etch holes (D).

$$\eta = \frac{A_h}{D^2} \tag{5}$$

For an isotropic sacrificial layer release process, the distance between etch holes also equals the minimum undercut etch depth. The parameter, η, can then be computed for a given geometry and spacing of etch holes in a structure. The most common configuration for most micromechanical structures is square etch holes that are diagonally separated by a distance equal to twice the minimum undercut etch depth. In this case, we can write down expressions for the effective Young's Modulus (E), density (ρ) and resonant frequency (f) for in-plane motion as a function of η as follows [9]:

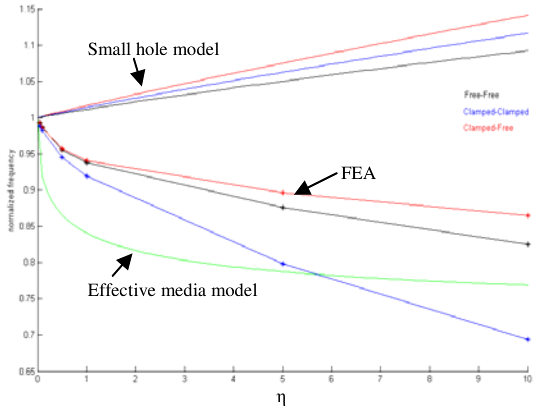
$$\frac{E_1^*}{E_s} = \frac{E_2^*}{E_s} = \frac{\sqrt{2}}{\sqrt{\eta}+\sqrt{2}} \tag{6}$$

$$\frac{\rho^*}{\rho_s} = \frac{\left(\sqrt{\eta}+\sqrt{2}\right)^2 - \eta}{\left(\sqrt{\eta}+\sqrt{2}\right)^2} \tag{7}$$

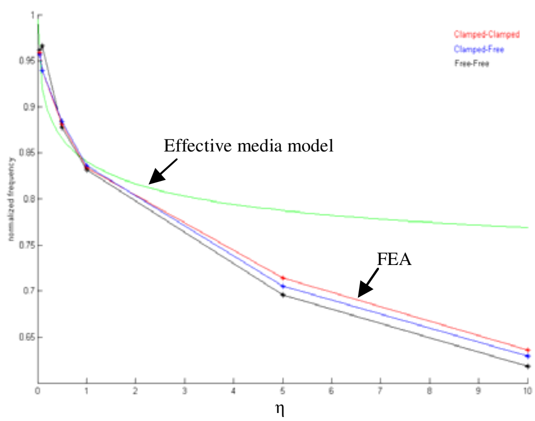
$$\frac{f^*}{f_s} = \sqrt{E^*/E_s}\Big/\sqrt{\rho^*/\rho_s} = \sqrt{\frac{\sqrt{2}\left(\sqrt{\eta}+\sqrt{2}\right)}{\left(\sqrt{\eta}+\sqrt{2}\right)^2 - \eta}} \tag{8}$$

Here the subscript, s, denotes the nominal value of the parameter. The change in Young's Modulus is assumed to be proportional to the relative distance between the etch holes while the change in density is assumed to be proportional to the relative area of the structures not covered with etch holes. Results from the small hole model and the effective media model were compared for beam-like structures with a large number of holes as the parameter, η, was varied. The results are shown in Figure 4. It can be seen that the small hole model is more applicable when η is small while the effective media model is more applicable while η is large.

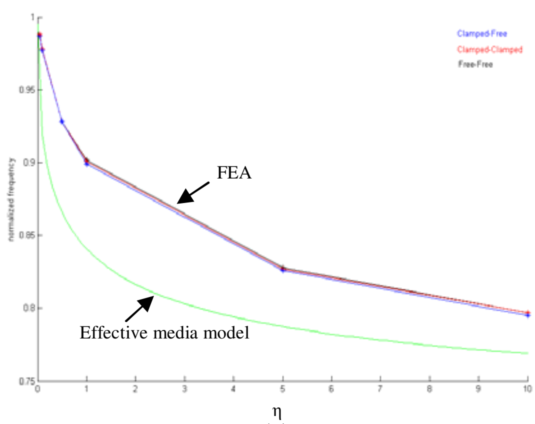
We also investigated the geometry of etch holes, their spacing and arrangement across the beam. Circular, triangular and hexagonal etch hole patterns were considered and compared with the square holes for fixed process minimum undercut depth, D. Hexagonal etch holes arranged in a triangular pattern can be shown to release more area relative to holes of other geometry for an isotropic sacrificial release process. Effective Young's Modulus and density can be calculated for each of these cases and compared to show that the structures with hexagonal holes introduce the least change in relative hole area and resonant frequency in this approximation.



(a)



(b)



(c)

Figure 4: A comparison of the small hole model and the effective density model with results from Finite Element Analysis. Normalised Frequency is plotted versus process parameter, η, for the first transverse mode (a), first torsional mode (b) and the first longitudinal mode (c). Clamped-clamped (red), clamped-free (blue) and free-free (black) characteristics are plotted for the given beam geometry.

4 CONCLUSION

Two analytical models have been used to predict the effect of etch holes on the resonant frequency of beam-like structures. The small hole model is based on the Rayleigh-Ritz principle for resonant frequency estimation and is useful in the limit where the density and number of etch holes is small. The effective media model is more useful in the limit when the density and number of etch holes is large. These models are used to examine the effect of the distribution of etch holes and the variation of hole topology on resonant frequency and the results are compared with Finite Element Analysis.

5 ACKNOWLEDGMENTS

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REFERENCES

[1] Nguyen, C-TC, "Vibrating RF MEMS for Low Power Communications", MRS Fall Meeting, Boston, MA, 2002, pp. J.12.1.1-J.12.1.12.

[2] Seshia, AA, Palaniapan M, Roessig, TA, Howe RT, Gooch, R, Schimert, T and Montague, S, "A Vacuum Packaged Surface-Micromachined Accelerometer", J. Microelectromechanical Systems, vol. 11, no. 6, Dec. 2002, pp. 784-793.

[3] Kaajakari, V, Mattila, T, Oja, A, Seppa, H, "Nonlinear Limits for Single Crystal Silicon Microresonators", J. Microelectromechanical Systems, vol. 12, no. 12, no. 5, pp. 714-725.

[4] Sharpe WN Jr, Vaidyanathan R, Bin Yuan, Gang Bao, Edwards RL. "Effect of etch holes on the mechanical properties of polysilicon," J. Vacuum Science & Technology B, vol.15, no.5, pp.1599-603. , Sept. 1997.

[5] Rabinovich VL, Gupta RK, Senturia SD. "The effect of release-etch holes on the electromechanical behaviour of MEMS structures," Tranducers 97. Part vol.2, pp.1125-8 vol.2. USA, 1997.

[6] Zou J, Balberg M, Byrne C, Chang Liu, Brady DJ. "Optical properties of surface micromachined mirrors with etch holes," J. Microelectromechanical Systems, vol.8, no.4, Dec. 1999, pp.506-13.

[7] Fang X, Myung N, Nobe K, Judy JW. "Modeling the effect of etch holes on ferromagnetic MEMS. IEEE Transactions on Magnetics," vol.37, no.4, pt.1 , pp.2637-9 , July 2001.

[8] Thomson W, "Theory of Vibration with Applications," Chapman & Hall.

[9] Gibson, LJ and Ashby, MF, "Cellular solids," Cambridge University Press, 1997.