Analytical Model for the Pull-In Time of Low-Q MEMS Devices

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ABSTRACT

A meta-stable transient region just beyond pull-in displacement that ultimately governs the pull-in time in critically damped systems is identified in this paper. Since the pull-in displacement time is basically governed by this second region (almost 90% of the pull-in time), the modeling of this region largely determines the reliability of the overall calculation of the pull-in dynamic transition. An analytical model for this region is derived and compared with measurements. The model accuracy, despite its simplicity, makes it a valuable tool for the design of MEMS switches outside vacuum and for sensors based on measuring pull-in time.

Keywords: Pull-in, MEMS dynamics, squeeze film damping, large-signal analysis

1 INTRODUCTION

Efficient and fast models for anticipating the dynamic and large-signal behavior of Microelectromechanical systems (MEMS) in the design phase are a topic of on-going research. The driver is the reduced design time and the increased probability of a first-time-right design. Complications are due to the various physical phenomena acting on MEMS.

A key characteristic of MEMS is the coupling between the different energy domains (mechanical, electrical, thermal) at the microscale level. Associated with the coupling between the mechanical and electrical domains, is the pull-in phenomenon. This instability in parallel plate electrostatic actuators has been subject of various studies [1,2]. If a quasi-static regime is assumed, mass and damping are neglected. The model describing the displacement of the MEMS structure in the direction normal to the electrode area, due to voltage applied, reduces to finding the equilibrium between mechanical and electrostatic forces. This results in a sudden pull-in at a well-defined pull-in voltage at 1/3 of the gap for 1 degree-of-freedom displacement structures [2].

A key issue in the design of MEMS-based switches and sensors based on pull-in time measurements is the dynamics of pull-in. In this case, the static regime does not apply, and for a meaningful study of the dynamic pull-in behavior, damping forces and mass inertia need to be included in the modeling. Some studies have considered the dynamic effects to enhance understanding of the pull-in time [3], or proposed a pressure sensor based on measuring the pull-in time [4]. More recently the pull-in time was shown to be sensitive to acceleration [5].

Studies on the dynamics of pull-in have generally assumed a smooth pull-in displacement. A more careful analysis reveals a meta-stable deflection for low-Q devices just beyond pull-in displacement. The pull-in time of realistic devices is determined by this particular region. It is therefore of crucial importance in the study of the dynamics of pull-in in general. Moreover, it is required for the design of the pressure sensor and accelerometer described, and for MEMS switches operated outside vacuum. This paper identifies the meta-stable regime. An analytical model is derived and compared with measurement results that confirm the modeling.

2 PULL-IN ANALYSIS

The structure used for analysis and experimental verification is basically a laterally movable beam with folded beam suspension at both ends and electrodes extending perpendicular to the axial direction. Three sets of stator electrodes in the same plane are used. One for electrostatic actuation in the direction normal to the electrode area and two sets of electrodes are used for capacitive displacement measurement (Fig. 1).

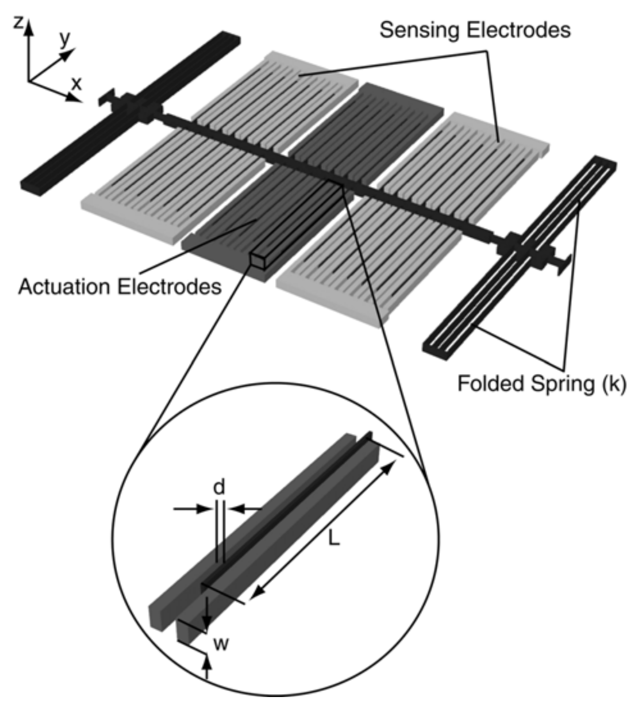

Figure 1: Drawing of the microstructure used.

The displacement of a MEMS device with 1DOF (Fig. 1) subject to an external acceleration and an input voltage is described by the differential equation:

$$m\frac{d^2x}{dt^2}+b(x,\frac{dx}{dt})\frac{dx}{dt}+kx=ma_{ext}+F_{elect} \tag{1}$$

Here m represents the movable mass, b the (nonlinear) damping coefficient, k the spring constant, and $F_{elect}=\frac{C_0 d_0 V^2}{2(d_0-x)^2}$ is the electrostatic force due to the voltage V applied across a capacitor C_0 with initial gap d_0. At around critical damping (Q≈ ½) the displacement x for an applied voltage V higher than the static pull-in voltage, $V_{pi}=\sqrt{\frac{8}{27}}d_0\sqrt{\frac{k}{C_0}}$ [2] ($V=\alpha V_{pi}$,with $\alpha>1$), proceeds as shown by curve in Fig. 2.

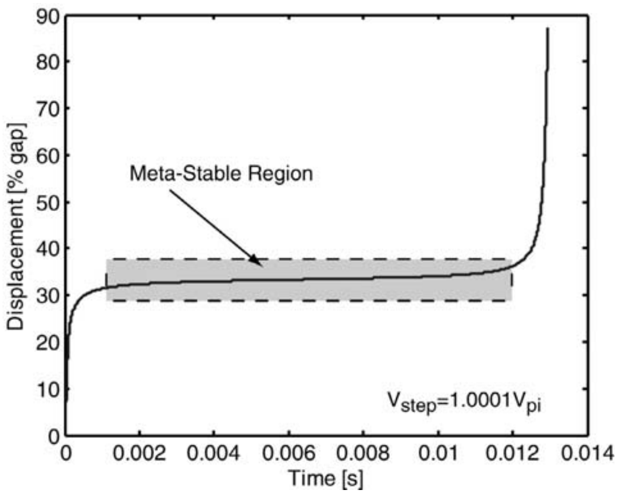

Figure 2: Pull-In displacement characteristic of low-Q (overdamped) microdevices.

2.1 Meta-stable Region

A qualitative analysis of the graph for the low-Q device, enables to distinguish three regions: a first region where the structure moves fast until close to the static pull-in displacement, a second meta-stable region where the movement is very slow, and finally a third region that takes the structure to the stoppers. Generally the first part of the curve is extrapolated to full deflection.

The fact that is often overlooked is that many realistic structures operate in the low-Q (Q< 1) mode and the curve presented in Fig. 2 applies. In such a system, the first region is where the electrostatic force imposed is compensated for by the mechanical and the damping forces. At the start of the step response the damping force dominates, but with deflection the mechanical force increasingly compensates for the electrostatic force. At the onset of the second region the structure moves very slowly and the mechanical force is almost the same as the electrostatic. This results in a kind of meta-stable equilibrium. Finally, due to the non-linear nature of the electrostatic force, the mechanical force cannot indefinitely compensate for the electrostatic force and the structure snaps.

The damping force is the crucial element in the first region, and largely determines the duration of the meta-stable region. Using a large-displacement dynamic model [6], the effect of the damping on the dynamic displacement was checked, and the different curves are presented in Fig. 3.

It is interesting to see that there is a threshold for which the meta-stability exists (at Q_{max}≈ 1.4). This is of crucial importance for understanding the mechanisms causing the meta-stable region, and can be of help when pull-in time is a crucial factor.

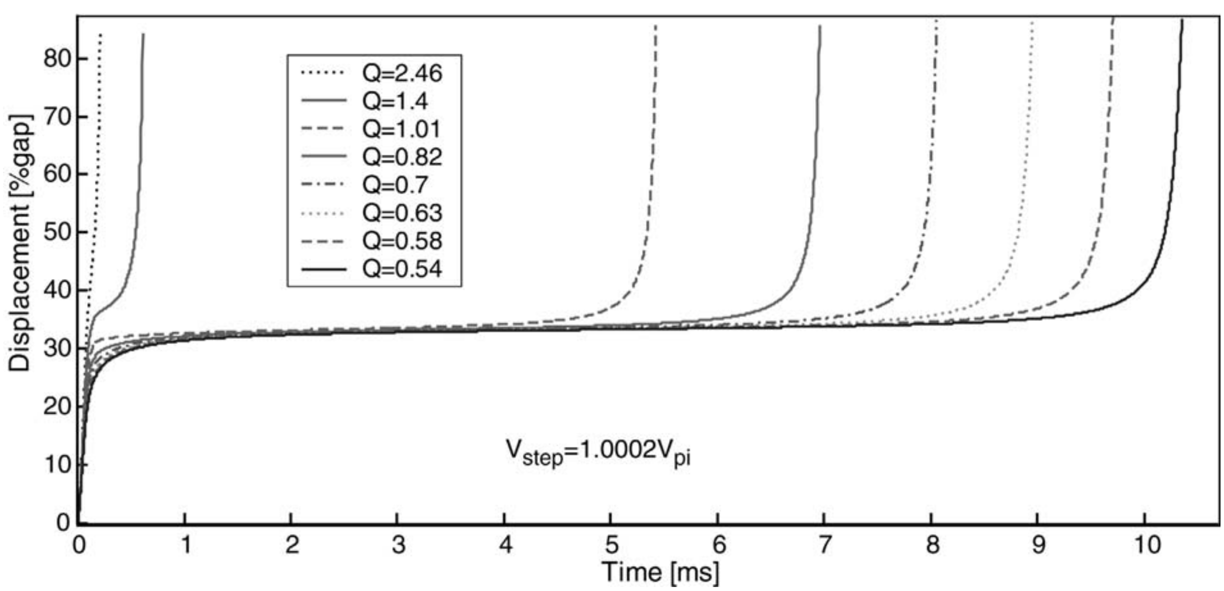

Figure 3: Simulations of the dynamic pull-in displacement for different Q values.

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3 MODELING

The previous section shows that the pull-in displacement time for a low-Q device is basically governed by the second region (almost 90% of the pull-in time), thus the modeling of this region only can be assumed to reasonably predict the pull-in dynamic transition.

The meta-stable region occurs around a small region around the static pull-in displacement $x_{pi} = d_0/3$. If a local linearization of (1) around x_{pi} is realized, a linear second-order system results. The two non-linear terms being linearized are the electrostatic force, and the damping coefficient.

The electrostatic force is approximated in Taylor series around x_{pi}:

$$F_{elect} = \frac{9C_0V^2}{8d_0} + \frac{27C_0V^2(x - x_{pi})}{8d_0^{\,2}} \tag{2}$$

3.1 Damping Coefficient

For structures in which only the size of the small gap between two plates changes in time, the pressure changes relative to the wall velocity are described by the Reynolds equation. An analytical solution for the forces acting on the surfaces can be found if some conditions are assumed [7]. The solution is frequency dependent and is not suitable for transient analysis. A very suitable approach is presented in [7] where the damping force can be represented by a network of frequency independent spring-damper elements, which have the same transfer function of the initial solution:

$$b_{m,n} = \frac{768lw\eta}{\pi^6 d^3 Q_{pr}} \frac{1}{(mn)^2\left(\frac{m^2}{w^2}+\frac{n^2}{l^2}\right)} \tag{3}$$

and:

$$k_{m,n} = \frac{64lwp_a}{\pi^4 d}\frac{1}{(mn)^2}, \tag{4}$$

where m and n are odd integers, d is the gap, η is the viscosity of the medium, P_a the ambient pressure and w and l are the width and length of the surfaces (Fig. 1), respectively. Q_{pr}, the relative flow rate coefficient is a function of the Knudsen number K_n, the ratio between the mean free path of the gas molecules and the gap separation. In this work, the flow rate coefficient is given by [7]:

$$Q_{pr} = 1 + 9.638\left(K_n\right), \quad K_n = \frac{\lambda_0 P_0}{P_a d}, \tag{5}$$

where λ_0 is the mean free path at pressure P_0.

If end effects are included [8], the frequency response of Fig. 4 is obtained for the gas film. For low frequencies the damping force depends linearly on the velocity of the plate (frequency), so that the film acts as a pure damper. At higher frequencies, the relation becomes non-linear and the spring force increases, indicating that at these frequencies the film is acting like a spring.

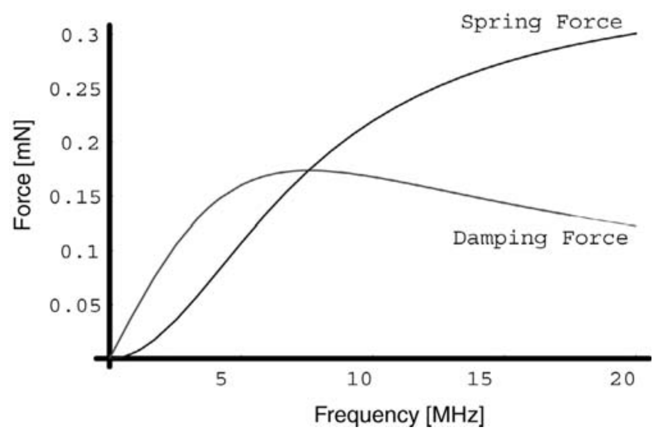

Figure 4: Damping and spring forces of the gas film.

For the analysis of the meta-stable region, the spring effect of the gas is neglected because the movement is very slow (low-frequency movement). The damping coefficient is linear in the frequency range of interest and can be computed for a gap size of $\frac{2d_0}{3}$.

3.2 Pull-In Time

Linearization of equation (1) around x_{pi}, while applying the variable transformation, $y=x-x_{pi}$ and assuming $V=\alpha V_{pi}$, gives the transfer function for the meta-stable regime:

$$H(s) = \frac{a_{ext} + \frac{kd_0}{3m}(\alpha^2 - 1)}{s^2 + \frac{b}{m}s + \frac{k}{m}(1-\alpha^2)} \tag{6}$$

For $\alpha>1$ the linearized transfer function presents two poles (a positive and a negative). As the dominant one is the positive pole (the system is unstable), the effect of the negative pole can be neglected without introducing a big error. With this simplification, an expression for the transition time, during which the structure moves from an initial starting point x_1 (start of the meta-stable region) to a final point x_2 (end of the meta-stable region) is given by the following formula ($\Delta x = x_2 - x_1$):

$$t_{pi} = \frac{Log\left[\frac{\left(k\left(\alpha^2-1\right)\left(d_0+3\Delta x\right)+3ma_{ext}\right)^2}{\left(d_0k\left(\alpha^2-1\right)+3ma_{ext}\right)^2}\right]\left(b+Root\right)}{4k(\alpha-1)(\alpha+1)},$$

$$with\,Root = \sqrt{4km\left(\alpha^2-1\right)+b^2} \tag{7}$$

4 EXPERIMENTAL RESULTS

Measurements of the pull-in time for different voltages and external accelerations have been performed on a surface micromachined accelerometer fabricated using an epi-poly process [9] (Fig. 5).

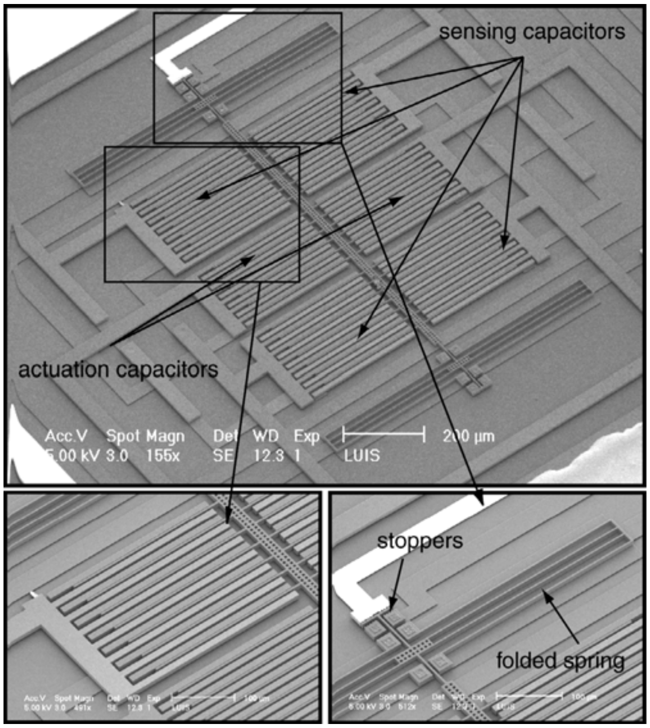

Figure 5: Photo of the fabricated device.

Fig. 6 compares the experimental results with the predictions based on (7). The important aspect is the high sensitivity of the transition time to an external acceleration. In fact, as the equilibrium of forces is the best description of the meta-stable region, it is intuitive that any small change acts as a perturbation of that meta-stable equilibrium, thus providing a means for achieving a very high sensitivity. This is valid only for small values of α.

5 CONCLUSIONS

In this paper a dominant region during dynamic pull-in displacement for low-Q devices has been identified and modeled. This meta-stable equilibrium region is very sensitive to external mechanical forces.

The meta-stable region has been studied using a simple linearized model. Since the dominant region occurs around a well-known displacement, linearization of the second-order equation of motion allows the derivation of a simple analytical expression for the pull-in time.

The model is confirmed by experiment. The model simplicity makes it a valuable tool for the design of MEMS switches operating outside vacuum, as well as accelerometers based on measuring the pull-in transition time.

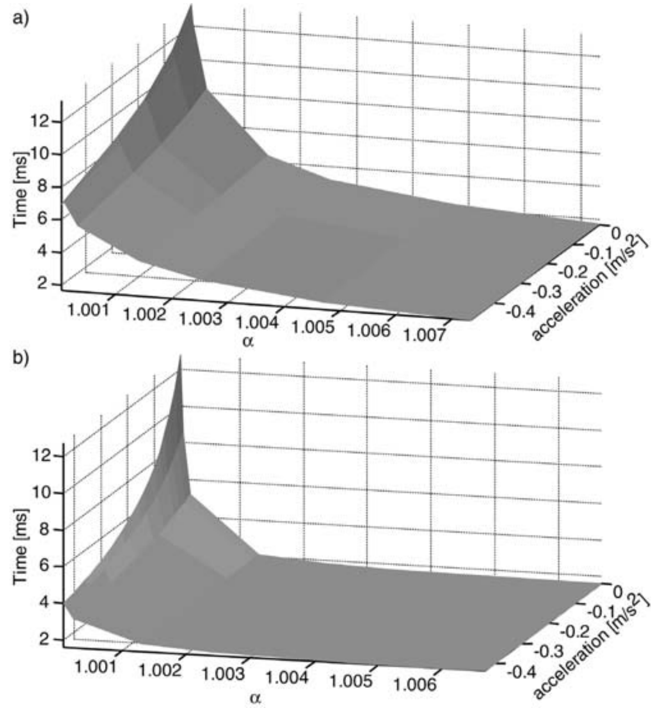

Figure 6: : Pull-in time changes with voltage step amplitude (α) and external acceleration. a) Experimental and b) Linear model.

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