A Full-System Dynamic Model for Complex MEMS Structures

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ABSTRACT

For full characterization of a surface micromachined MEMS device, where the thickness of the moving layer is just a few times the gap size, the modeling has to take large-signal behavior and end effects into account. In this work, a numerical method using finite differences is implemented in Simulink to solve the Reynolds equation. The spatial derivatives are solved using the finite differences. The use of the Simulink capabilities for time integration allows solving of the time derivatives at any mesh point. To increase efficiency, a low-level language is used inside Simulink to solve the parameterized finite differences and the time derivatives. As the Reynolds equation is being solved inside a high-level language description, other system parameters (such as: mass, spring constant, non-trivial geometry) can be easily incorporated. Measurements on a complex 2DOF MEMS device are compared with simulation results, and the agreement validates the full system approach proposed.

Keywords: MEMS modeling, squeeze film damping, large-signal analysis, macro model

1 INTRODUCTION

The penetration of microelectromechanical system MEMS technology in an increasing number of applications calls for advanced modeling tools to deal with the complexity of the system on the microscale and the commercial drive towards first-time-right and fast turnaround design [1].

The dynamics of a MEMS microstructure is governed by inertia and the squeeze-film damping. The non-linear behavior of the Reynolds equation and the frequency dependence of the gas film present a modeling challenge.

For simple geometries and simple movements, the Reynolds equation can be solved and efficient full-system models based on analytical solutions can be implemented [2,4] accounting for large signal behavior [3,4] and including end effects [5,6]. For more complex geometries, the Reynolds equation can not be solved analytically and numeric solutions have to be used. Usually these solutions are based on Finite Element Modeling [6], or Finite Differences [7]. However, for simulation at the full-system level, while accounting for large signal and end effects, the calculation effort required to accurately describe the motion of the overall system becomes excessive. In this paper we

report a flexible way to model complete MEMS systems. Reynolds equation is solved using the finite differences method, implemented in such a way that other relevant system parameters can be introduced. Simulations of a model of a 2-degree-of-freedom (2DOF) structure are compared with measurements.

2 MOMENT ACTUATED ACCELEROMETER

The device used in this work is a 2DOF moment-actuated accelerometer (Fig. 1). The movement of the structure is fully characterized by two state variables: displacement w_1, and angle φ_1 (Fig. 2). Such a moment actuated device may compare favorably to a normal 1DOF accelerometer in terms of damping coefficient (higher quality factor). The modeling difficulties of the structure arise from the fact that it presents two distinct movements (translational and rotational) with cross-couple terms in between.
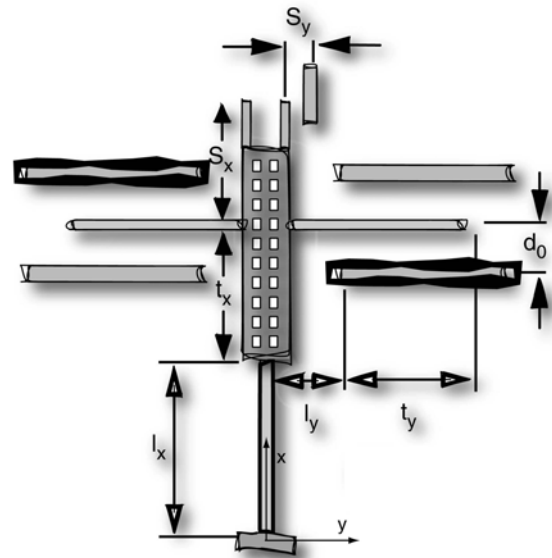

Figure 1: Schematic of the moment actuated accelerometer.

2.1 Fabricated Device

The epi-poly process was used for the fabrication of the test structures [8]. This process is very suitable for the fabrication of relatively thick and high aspect ratio free-standing beams on top of a silicon wafer. This device is

basically a free-standing lateral beam (200 μm long, 3 μm wide and depth of 10.6 μm) anchored at one end (the base) only (Fig. 3).
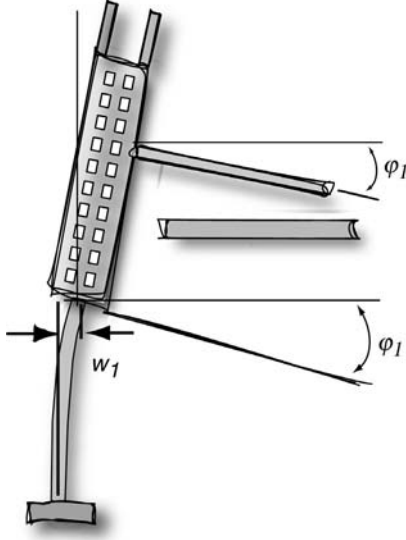

Figure 2: Identification of the state variables used in the 2DOF model.

The beam can be deflected by electrostatic actuation in the plane of the wafer using a voltage applied across parallel plate capacitors (2μm gap). These are composed of two sets of electrodes located alongside the free-standing tip, with counter electrodes anchored to the substrate. The deflection can be measured using a set of differential sense capacitors located alongside the free-standing tip. Finally, there are electrically isolated stoppers to limit the lateral motion.

3 SQUEEZE FILM MODELING

For structures in which only the width of the small gap between two plates changes in time, the pressure changes p relative to the wall velocity are described by the modified Reynolds equation [3]:

$$\frac{\partial}{\partial x}\left[\rho h^3 Q_{pr} \frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[\rho h^3 Q_{pr} \frac{\partial p}{\partial y}\right] = 12\eta \frac{\partial(\rho h)}{\partial t}, \qquad (1)$$

where ρp^{-n} is constant, and the pressure p, gas density ρ and gap size h are functions of space and time. The gas has a viscosity η, and Q_{pr} is the relative flow rate coefficient. When an isothermal process is assumed (n=1), density ρ can be replaced with pressure p.

The modified Reynolds equation is used, because rarefaction effects have to be included. For transitional and molecular damping regimes, Q_{pr} is a function of the Knudsen number K_n, the ratio between the mean free path of the gas molecules and the gap separation. In this work, the flow rate coefficient is given by [2]:

$$Q_{pr} = 1 + 9.638\left(K_n\right), \quad K_n = \frac{\lambda_0 P_0}{ph}, \qquad (2)$$

where λ_0 denotes the mean-free-path at pressure P_0.

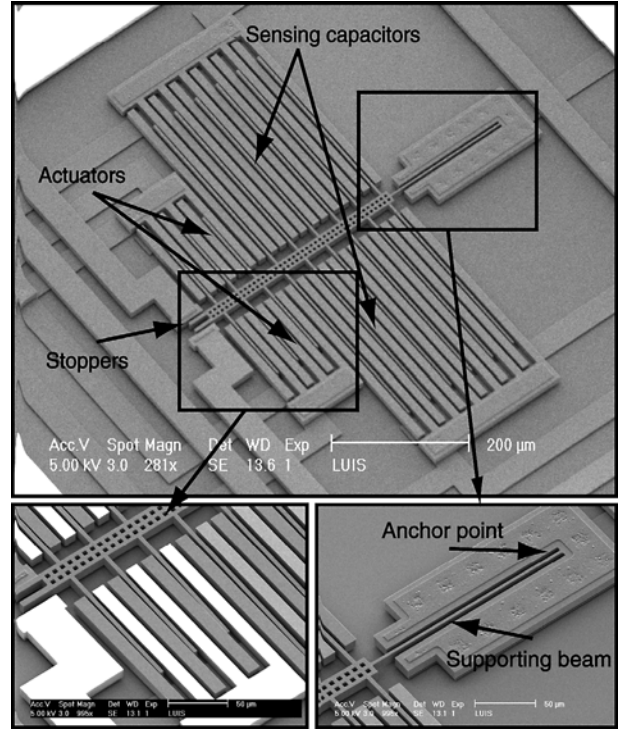

Figure 3: Photograph of the fabricated device.

3.1 Finite Differences Model

For the finite differences model, the surface is first divided into a rectangular grid of M×N elements (x=m.Δx, y=n.Δy, m=0…M-1, n=0…N-1). At each mesh point, equation (1) is implemented. The spatial derivatives are solved using the finite difference method [9]. Using this method, we end up with a set of M×N time differential equations.

Very important in any squeeze-film model, is the inclusion of large signal effects (already accounted for in the modified Reynolds equation) and end effects [5,6]. Incorporation of the end effects in this model implies that the pressure on the plate edges is not simply assumed at ambient pressure, but rather that the system dynamics are also considered at the device edges [6].

3.2 Model Implementation

In order to solve the time differential equations, Simulink was used. A parameterized model was built in a low level language (C language) and introduced in Simulink.
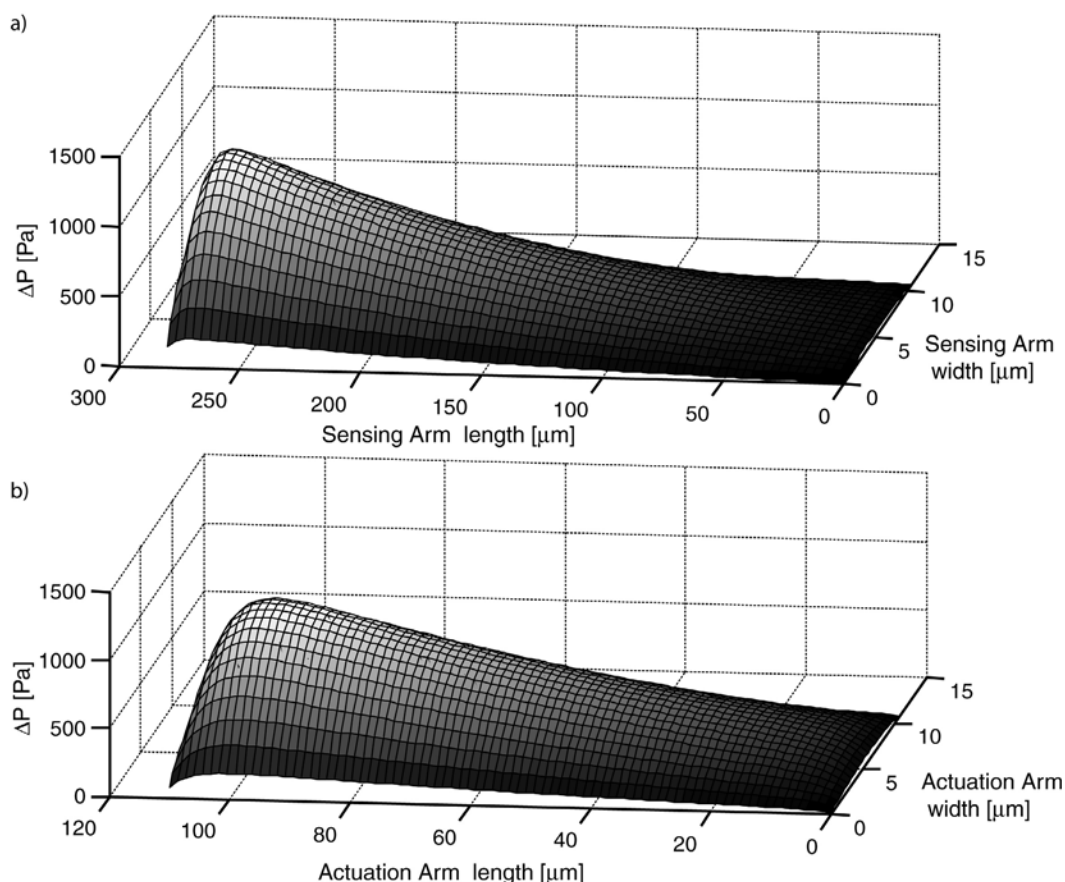
Figure 4: Pressure distribution in a a) sensing and b) actuation arm using a 80×20 finite difference mesh.

For each time interval, the spatial derivatives are solved using the finite difference method, and the time derivatives are solved by the methods already implemented in Simulink. The use of a high-level language description enables the introduction of other system properties (full system functionality), and a very good parameter flexibility. Once the model is implemented is very easy to study the influences of the various parameters (mesh size, structure dimensions, pressure changes, gap sizes, etc.).

The implemented squeeze film model was tested for the different arms of the structure (actuation and sensing arms). As these have different sizes, different mesh grids have to be used for each of the arms. The pressure distribution is presented in Fig. 4 for a single actuation and sensing arm, when the structure oscillates with a maximum angle (φ_1=0.0051 radians) and displacement (w_1=0.685μm) at 400 kHz.

3.3 Reduced-Order Model Generation

Another significant advantage of the use of a high level language description is the increase in flexibility. Reduced-order modeling techniques [10] have been introduced to solve dynamic problems. Based on some FEM or FD simulations, a reduced model can be build having the same response of the original gas film full model (even for complex geometries).

For testing the finite difference modeling approach and evaluation of the advantages of reduced-order models, a reduced-order model was build with just a spring-damper network [10]. For a large-signal behavior of the reduced-order model, simulations have to be performed for several gap sizes, since the values of the spring and damper are gap dependent. The huge advantage of the reduced model is a large decrease in computer time per simulation.

Moreover, the flexibility of the finite difference model enables the automatic implementation of the squeeze-film reduce model: the simulations are performed in a programmed sequence, and all the fitting that is needed is automatically generated.

After the generation of the reduced-order model, some results of the finite difference model are compared with the reduced model. Fig. 5 shows the damping moment of both models, when the structure oscillates with a maximum angle (φ_1=0.0051 radians) and displacement (w_1=0.685μm) at 500 kHz.

4 FULL SYSTEM MODEL

The movement of the 2DOF structure, in the absence of an external acceleration, can be described by the non-linear differential equations (a voltage V is applied to the actuation arms):

$$\begin{cases} F_m + F_b(w_1, \varphi_1) + F_k(w_1, \varphi_1) = F_{elect}(w_1, \varphi_1, V) \\ M_m + M_b(w_1, \varphi_1) + M_k(w_1, \varphi_1) = M_{elect}(w_1, \varphi_1, V) \end{cases} \quad (3)$$

A translation and a rotational equation describe the full system. Most of the forces and moments depend on both state variables (w_1 and φ_1) – cross-couple terms are present. All these dependencies can be easily implemented within Simulink. The full system (with the finite difference method used to compute the damping force and moment) is thus implemented. Simulations were performed for various input voltages.
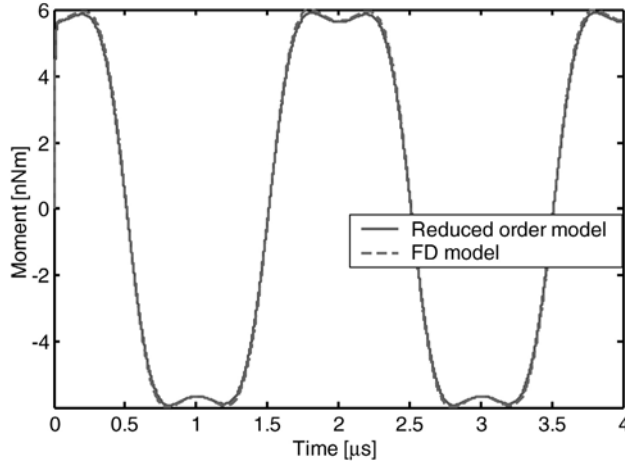

Figure 5: Comparison of the damping moment between full and reduced order models.

5 EXPERIMENTAL RESULTS

Measurements on fabricated devices have been compared with simulations (Fig. 6). The results are in good agreement, thus demonstrating the validity of the model.
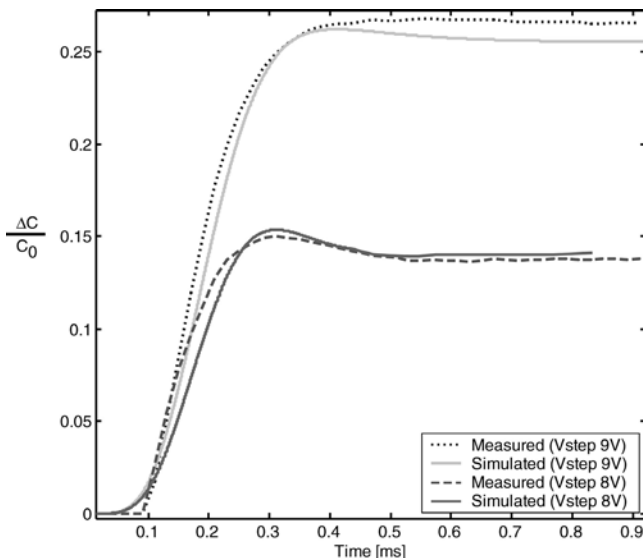

Figure 6: Measured and simulated capacitance change for an input step of 8 and 9 volts.

6 CONCLUSIONS

As demonstrated, MEMS devices can generally be modeled with Finite Differences. Even the motion of very complex geometries is adequately described by one translation – perpendicular to the gap – and two rotational movements.

The full-system can be simulated with this approach. The capability of automatic generation of reduced-order models is another advantage of the proposed model. As shown in Fig. 5, a simple spring-damper network presents the same behavior of the modified Reynolds equation. This may lead to a fast development time in the design phase, and also to a much better understanding of the full dynamics of complex MEMS structures.

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