Analytical and FEM Simulation Pull-in Study on Deformable Electrostatic Micro Actuators

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ABSTRACT

In this paper, we employed an easy general theory [1, 2] over FEM simulation to carry out the pull in analysis of cantilever beam, fixed-fixed beam and circular membrane actuators and compare the results with the FEM simulation. This analytical method utilizes capacitance-based generalized equations and provides an easy and time saving platform over complicated coupled FEM simulations for pull in analysis of complex system. It is found that deformation curves of these three electrode configurations are independent of geometry and voltage. It is shows that the three deformable actuators could reach travel ranges of 47.2%, 42% and 45.6% respectively. Fringe effect is also considered in this analytical method. The fringe effect will increase the travel range from 40% to 42% for fixed -fixed beam, from 45.4% to 47.2% for cantilever beam, but decreases the travel range from 45.6% to 41.6% for clamped circular membrane.

***Keywords**:* Micro-actuator, pull in, electrostatic, FEM, analytical, deformable

1. INTRODUCTION

With the recent growth of MEMS technology, electrostatic actuator has become one of the most common micro-mechanical components in developing complex micro-systems, for example, switching fabric for all optical DWDM systems. The actuation of an arbitrary actuator could be simulated by FEM analysis however it is complicated and time consuming. Analytical method is expected to simplify the prototyping design. Existing analytical method has been applied by balancing elastic restoring force with electrical attractive force, to construct structural equation and obtain pull in solution [3]. An easy and time saving general theory over complicated FEM simulation was proposed by our group [1, 2]. Using this theory, Full actuation of cantilever torsion actuators with different shapes and sizes was reported to have a normalized pull in position from 0.33 to 1.0 [2, 3]. Other actuators, especially with deformable proof mass, have been widely used for device developments, which makes it desirable to run their pull-in analysis not only to check the physics insight implicated in these actuation scheme, but also for the purposes of a convenience engineering reference. However, the systematic pull-in study on these electrostatic deformable micro actuators is absent.

This paper systematically investigated this topic by combining general capacitance theory and classical elastic beam theory. Analytical results are to be compared with FEM results. However, the method presented in this paper is not to be viewed as a replacement to the traditional methods for MEMS. It will provide an efficient support for prototyping design.

2. THEORY

An electrostatic actuator can be modeled as a variable capacitor suspended by elastic springs. The equations to describe this system are derived under the assumption that the spring stiffness, K, is constant and independent of the actuated strain.

Applying conservation of energy to the electro-mechanical system and differentiating with respect to displacement, *X*, results in a structural equation in the force domain given by:

$$\frac{1}{2}\frac{\partial C}{\partial X}V^2 = KX \tag{1}$$

where C is the variable actuator capacitance. When the gap is variable along *x* direction. The capacitance can be written as:

$$C = \int_0^L \frac{\varepsilon}{g_0 - y(x)} dA(x) \tag{2}$$

where L is the length of plate, g_0 is the original gap as shown in figure 1(a), (b), and (c). $y(x)$ is the displacement. Multiplying by *X* on both sides of Eq. (1), and differentiating with respect to *X*, leads to the pull-in equation for the structure when evaluated at the pull-in displacement position:

$$\left.\frac{\partial^2 C}{\partial^2 X} - \frac{1}{X}\frac{\partial C}{\partial X}\right|_{X = X_{pin}} = 0 \tag{3}$$

The pull-in voltage can be therefore derived by combining equation (1) and equation (3):

$$V_{pin} = \sqrt{2K \Big/ \left(\left.\frac{\partial^2 C}{\partial X^2}\right|_{X = Xpin}\right)} \tag{4}$$

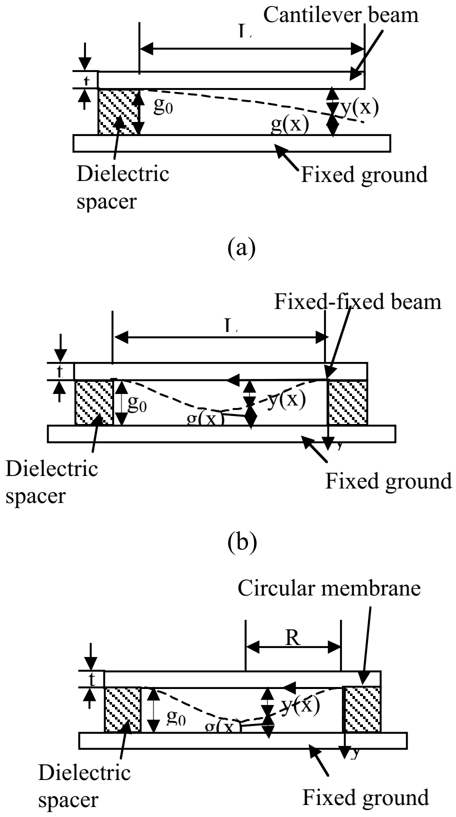
rewriting pull-in equation at pull-in point results in:

$$\frac{1}{2}\frac{\partial C}{\partial X}V_{pin}^2 = KX_{pin} \tag{5}$$

combination of equation (1) and (5) cancels out spring parameters and yield a two-parameter unitless equation:

$$\Lambda \bullet U^2 = 1$$

$$\Lambda = \frac{\frac{1}{X}\frac{\partial C}{\partial X}}{\frac{1}{X_{pin}}\left.\frac{\partial C}{\partial X}\right|_{X=X_{pin}}}, \quad U = \frac{V}{V_{pin}} \qquad (6)$$



(a)

(b)

(c)

Fig. 1 Schematic view of three deformable micro actuators (a) Cantilever beam, width=W; (b) Fixed-fixed beam, width=W; (c) Circular membrane with radius R

To solve the gap $g(x)=g_0 - y(x)$ as in equation (2), a two-dimensional pull-in model [4] was built based on well-known beam and plate theory. The summary of the governing equations for the electro-structural problem is listed in Table 1. The 2D model accounts for the fact that actual structures have non-rigid, position-dependent gaps. Also in the case of beams, a first order fringing field correction term is added to the electrostatic pressure, which further improves the accuracy of the 2D model. By representing the force per unit area as that of a locally parallel plate capacitor, this model assumes only small-angel bending and neglects any non-uniformity in electric filed due to curvature. This is a reasonable approximation for the structures discussed here because they all have gaps, which are small compared to their lateral dimensions.

Table 1: Summary of governing equations

	Governing equation
Cantilever beam	$\hat{E}I\frac{d^4g}{d^4x} = -\frac{\varepsilon_0 V^2 w}{2g^2}\left(1+0.65\frac{g}{w}\right)$
Fixed-fixed beam	$\hat{E}I\frac{d^4g}{d^4x} = -\frac{\varepsilon_0 V^2 w}{2g^2}\left(1+0.65\frac{g}{w}\right)$
Circular membrane	$D\nabla^4 g = -\frac{\varepsilon_0 V^2}{2g^2} + P$
	Note: $\hat{E} = \frac{E}{1-\nu^2}$, $D = \frac{Et^3}{12(1-\nu^2)}$

Note: biaxial residual stress in Fixed-fixed beam and Circular membrane is assumed to be zero.

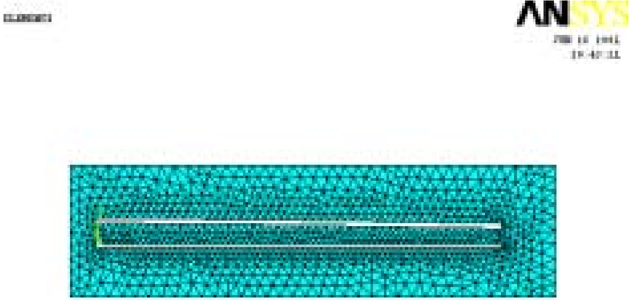
3. ANALYTICAL AND FEM SIMULATION

3.1 Deformation curve

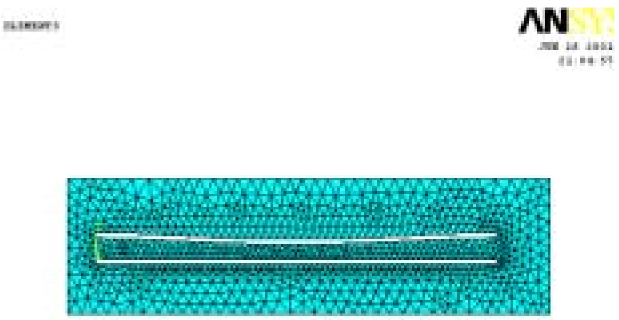
The deformation curve can be solved by commercial software package MATLAB. FEM simulation of this electrostatic-structural coupled problem is solved by commercial code ANSYS. Fig. 2(a) and (b) shows the configuration of FEM simulation results for (a) Cantilever beam and (b) fixed-fixed beam. The configuration of deformed circular membrane is similar to fixed-fixed beam, and was solved axisymmetrically.

Fig. 3 (a), (b) and (c) show the normalized deformation curves of the three actuators when a bias voltage is applied between electrode and proof mass, using both analytical method and FEM simulation. The deformation curve is defined as the ratio of displacement to the maximum displacement, y/y_0 .

The analytical method solves a 4th order ODE for each actuator and the deformation curve is obtained by curve fit of the analytical numerical results. There is a good agreement between the analytical and FEM simulation results. It is also found that the deformation curves, y/y_0 , are independent of geometry and voltages if fringe effect is small.

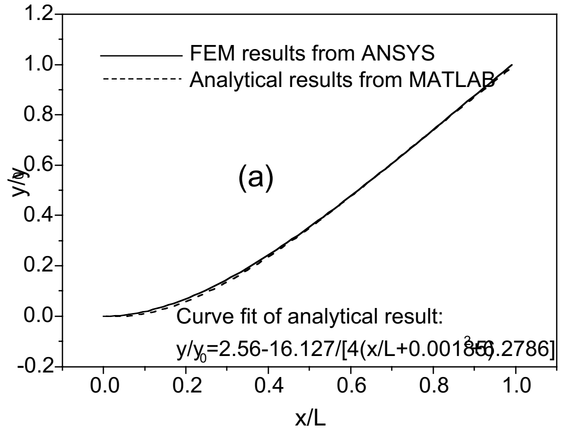


(a)

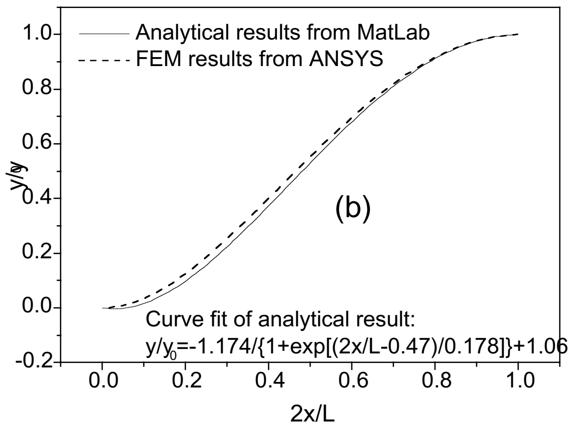


(b)

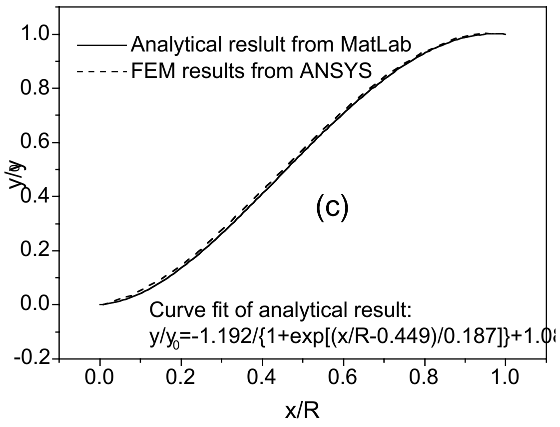
Figure 2. The configuration of FEM simulation results for (a) Cantilever beam and (b) fixed-fixed beam.



(a)



(b)



(c)

Fig. 3 Comparison of deformation curves between analytical method and FEM simulation (a) Cantilever beam; (b) fixed-fixed beam; (c) Circular membrane

3.2 Prediction of pull-in position and voltage

The deformation curve is applied to equation (2) to calculate the capacitances of the micro-actuation of three configurations. These capacitances are then utilized to predict the pull-in position using generalized-capacitance based theory (equations 3~6).

Normalized pull-in position, Θ, is defined as ratio of pull-in position, X_{pin}, to the original gap, g_0. The normalized pull-in analysis results are listed in Table 2. There is a good agreement between analytical and FEM simulation. It shows that the three deformable actuators could reach travel ranges of 47.2%, 42% and 45.6% respectively, different from 44.04% for commonly employed rectangular torsion actuators.

Table 2: Summary of results of normalized pull-in positions from analytical and FEM simulation

	Normalized pull-in position	
	Analytical	Simulation
Cantilever beam	45.4~47.2%	45~47%
Fixed-fixed beam	40.4~42%	40~42%
Circular membrane	41.6~45.6%	41~45%

The pull in voltage can also be predicted by this analytical method. After X_{pin} is solved, the second partial derivative $\left.\frac{\partial^2 C}{\partial X^2}\right|_{X=Xpin}$ can be calculated and then the pull-in voltage is calculated by equation 4,

$V_{pin} = \sqrt{2K \Big/ \left(\left.\frac{\partial^2 C}{\partial X^2}\right|_{X=Xpin} \right)}$. Fig 4. shows that there is a good agreement of pull-in position and voltage between analytical method and FEM simulation for fixed-fixed beam.

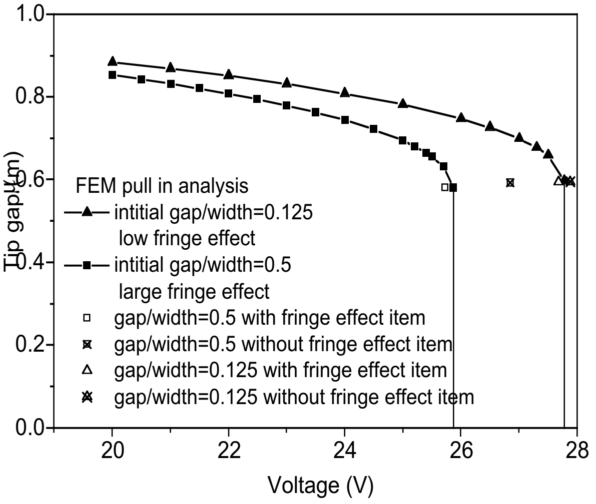


Figure 4. Comparison of pull-in position and voltage between analytical method and FEM simulation, with and without fringe effect, for fixed-fixed beam

3.3 Fringe effect

Fringe effect is also considered in this paper. With fringe effect, the governing equation is changed therefore deformation curve and the pull-in position will also be changed. The comparison of pull-in position and voltage with and without fringe effect for fixed-fixed beam is also shown in figure 4. The FEM simulation and analytical result are in good agreement as well. Fringe effect on pull-in position and voltage for fixed-fixed beam has been investigated analytically using the general theory and the result in shown in figure 5.

The fringe effect will increase the travel range from 40% to 42% for fixed-fixed beam. The fringe effect on travel range for cantilever beam and circular membrane has also been studied using analytical and FEM simulation. It is found that the fringe effect increases the travel range from 45.4% to 47.2% for cantilever beam. However, fringe effect decreases the travel range from 45.6% to 41.6% for clamped circular membrane as shown in figure 6. The summary of main results is listed in Table 1.

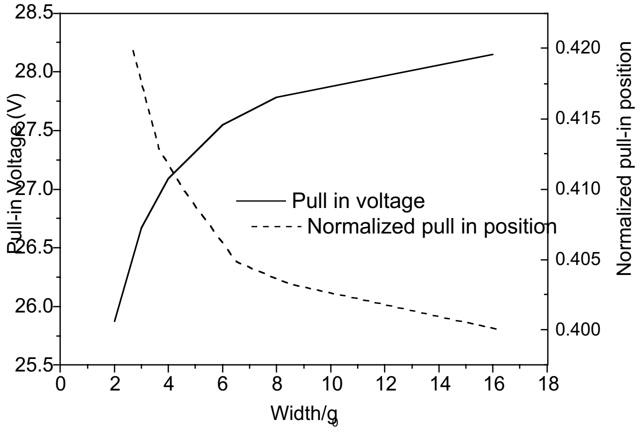


Figure 5. Effect of fringe effect on pull-in position and voltage predicted by analytical method, for fixed - fixed beam

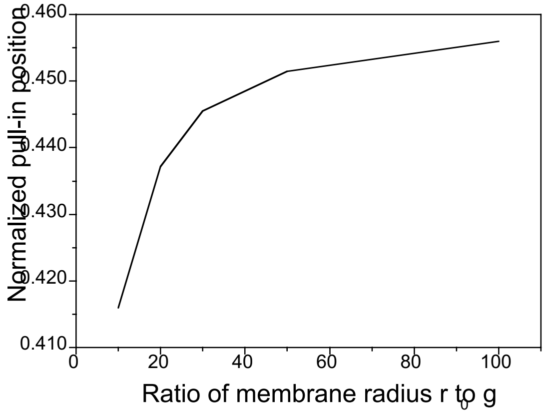


Figure 6. Effect of fringe effect on pull-in position predicted by analytical method for circular membrane.

CONCLUSIONS

This paper systematically addressed the pull-in behavior of deformable electrostatic micro-actuators using capacitance based analytical method and FEM. Pull-in behavior of actuators by electrodes in three various configurations was discussed in detail. Fringe effects are also considered in pull-in analysis. It is proved that this analytical method is valid in predicting pull-in behavior of deformable electrostatic micro-actuators.

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